Uncertain safety:
textile-strengthened reinforced concrete structures

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1 Introduction

Consistently occuring failures of buildings clarify that an absolute safety does not exist. However, the task of engineers is it to plan and build in such a way that only a very small risk arises. This just accepted risk is specified in technical standards. For the definition the security of humans and goods, which are endangered by collapsing buildings, is crucial.

In order to ensure a sufficient safety, different concepts have been developed, whose aim is the evaluation of safety. Safety is not measurable, but a subjective perception, which can be individually different. Therefore the degree of safety (the safety level) is expressed by qualitatively different values, e.g. by a safety factor or the failure probability.

The safety level of a building is not constant over its lifetime, but time-dependent. For example damage, fatigue or use for new purposes can lead to the reduction of the safety level. It must be the aim to detect early enough if the safety level falls under the accepted limit in order to avoid cases of damage. With strengthening (e.g. with textile-reinforced finegrade concrete layers) and maintenance measures the safety level can be raised again into the range of the accepted risk.

The building "Erlweinspeicher Dresden" (built 1913 to 1914), represented in Fig. 1, is to be converted to the congress hotel. It is the first large building in reinforced concrete construction way in Dresden. The general impression of the building, caused by the strongly damaged front, tempts to the subjective statement that the structural safety is no longer sufficiently ensured. Is this impression justified? This question can only be answered with an appropriate quantification of the present safety level under consideration of history (e.g. loading process, fire loading at the end of the Second World War). For the future use prognoses must be provided. If the prognosticated safety level is placed under the accepted limit, load-bearing structural elements must be strengthened and/or repaired. For the modified structure the safety level must be assessed again under consideration of the modification of the system.

Figure 1: Erlweinspeicher Dresden

Safety is ensured, if the effect of actions (stress S) is smaller than the stressability (system resistance) R of a structure

$$S < R \tag{1}$$

Loads, internal forces, and tensions belong to the stress S. System resistances are point failure tensions, cross section failure forces, and system failure loads depending on the respective failure mode.

If we assume that loads, geometry, and material parameters are deterministic values, then also R and S are deterministic. Using a global safety factor Eqn. (1) can be written in the form

$$\nu \cdot S \leq R \tag{2}$$

This classical, deterministic safety concept is still used in the present. In some disciplines of the civil engineering it has already been replaced by an improved safety concept based on partial safety factors.

The safety statement of the deterministic safety concept is not differentiates enough. Moreover, loads, geometry and material parameters are not deterministic values. Their current values vary; they are uncertain. For example the compressive strength of several concrete cylinders after 28 days will always be different.

If we regard the current values of loads, geometry and material parameters as result of a random event, we can introduce these parameters as random variables. The resistance R and the stress S are then likewise random variables.

The failure probability may be then determined with the aid of the probability theory

$$P_f = P\,(R < S) \tag{3}$$

$$P_f \leq \text{perm}\ P_f \tag{4}$$

The failure probability P_f expresses the probability P that R< S holds. This is a measure for safety. It is demanded that P_f is less than or equal to a permissible failure probability perm P_f. This perm P_f is the accepted limit for cases of failure, it has been calibrated on the basis of different causes of damage and death respectively and may be found today as reliability index β in the standards (e.g. EC 1, DIN 1055-100).

Frequently, samples with a limited number of sample elements are only available for uncertain parameters, from which a formal mathematical description of the uncertainty must be developed. The test theory of classical statistics permits testing of a sample for randomness (see ch. 2.1). If the sample does not exhibit the property of randomness, other uncertainty models such as, e.g. fuzzy randomness, must be adopted (see ch. 2.2).

The assessment of the reliability - as complement to the event failure - is the subject of the structural design. The reliability is significantly determined by the mechanical structural model (linear, physical nonlinear, geometrical nonlinear). A condition for the execution of a non-deterministic safety assessment (ch. 3) is therefore always a close-to-reality deterministic solution.

The development of the uncertainty evaluation in the context of the structural analysis can be approximately characterized by the statement

"from the safe uncertainty to the uncertain safety".

2 Uncertain data

The uncertainty of the safety is considerably caused by the uncertainty of the parameters. For the safety assessment these parameters must be quantified. The method which may be used for quantification is to be selected as a function of the cause of the uncertainty.

Causes for uncertain parameters may be:

- Only a sample of limited size is available.
- Geometry, material parameters and loads vary - they are uncertain in their perception.
- Sampling and experiments for the characterisation of data take place under unknown, non-constant reproduction conditions.

Well known data models, which are used for quantification, are based on deterministic variables (safety factors), interval variables, or random variables. By means of a data analysis it can be verified whether the data can be appropriatly described with these models.

2.1 Data analysis

For the specific sample shown in Fig. 2 an assessment must be made in relation to the uncertainty to be assigned. The sample elements describe the tensile strength of glass filament yarn.

The sample is evaluated on the basis of estimation and test theory. In order to examine the random properties of the sample shown in Fig. 2, non-parametric tests are carried out. The zero hypothesis H_0, "the sample elements are random" , is rejected with a level of significance of 0.987. The test of homogeneity for the zero hypothesis H_0, "the two sub-samples containing elements 1-55 and 56-110 respectively, are derived from the same distribution", is rejected with a significance level of 1.0. With the aid of goodness-of-fit tests, the type of probability distribution is sought. The KOLMOGOROV SMIRNOV goodness-of-fit test yields a rejection certainty of 0.0 for the normal distribution as well as the logarithmic normal distribution. The unique assignment of a distribution type is hence not possible. The i.i.d. paradigm (i.i.d. = identically independently distributed) of statistics is not fulfilled.

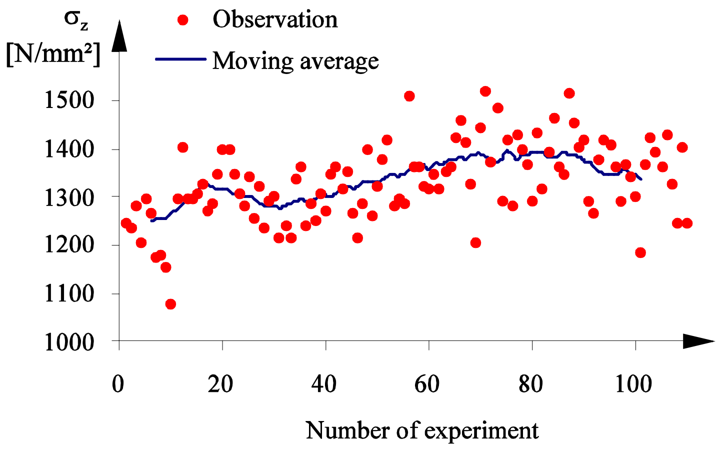


Figure 2: Tensile strenght of glass filament yarn NEG-ARG 620-01

Informal uncertainty, e.g. for defining the distribution type of a universe, does not always permit a description of uncertainty by randomness alone. This is illustrated by a numerical study. A universe is considered with a theoretically exact normal distribution with the parameters $\mu = 3$ and $\sigma^2 = 1$. 40,000 samples, each containing n = 100 elements, are considered. Although the distribution type for the universe is known, a normal distribution, a logarithmic normal distribution, and a GUMBEL distribution are assumed as alternatives for the chosen samples. For each sample the parameters of the assumed normal distribution, logarithmic normal distribution, and GUMBEL distribution are estimated by means of the maximum likelihood method. The expected values E(X) and the variances VAR(X) of the 40,000 estimated distributions are computed and shown in Fig. 4 in the form of a cumulative frequency distribution curve. Although the samples are drawn under constant conditions, it is apparent that significant informal uncertainty exists when the distribution type of a universe cannot be correctly determined.

Particularly in civil engineering, it cannot be presumed that measurements for determining parameters (drawing of samples) are carried out under constant conditions. Material, geometrical, and loading parameters are affected by a multitude of variable factors. This leads to non-compliance with the underlying i.i.d. paradigm. Even when applying BAYES statistics as a possible means of introducing additional information via a universe, the fulfillment of the i.i.d. paradigm for the sample variables of several samples is disputed. For it is assumed that the distribution of the universe, from which different samples are sequentially drawn, remains constant. The assumed a priori distribution may significantly affect the results and may lead to non-negligible errors.

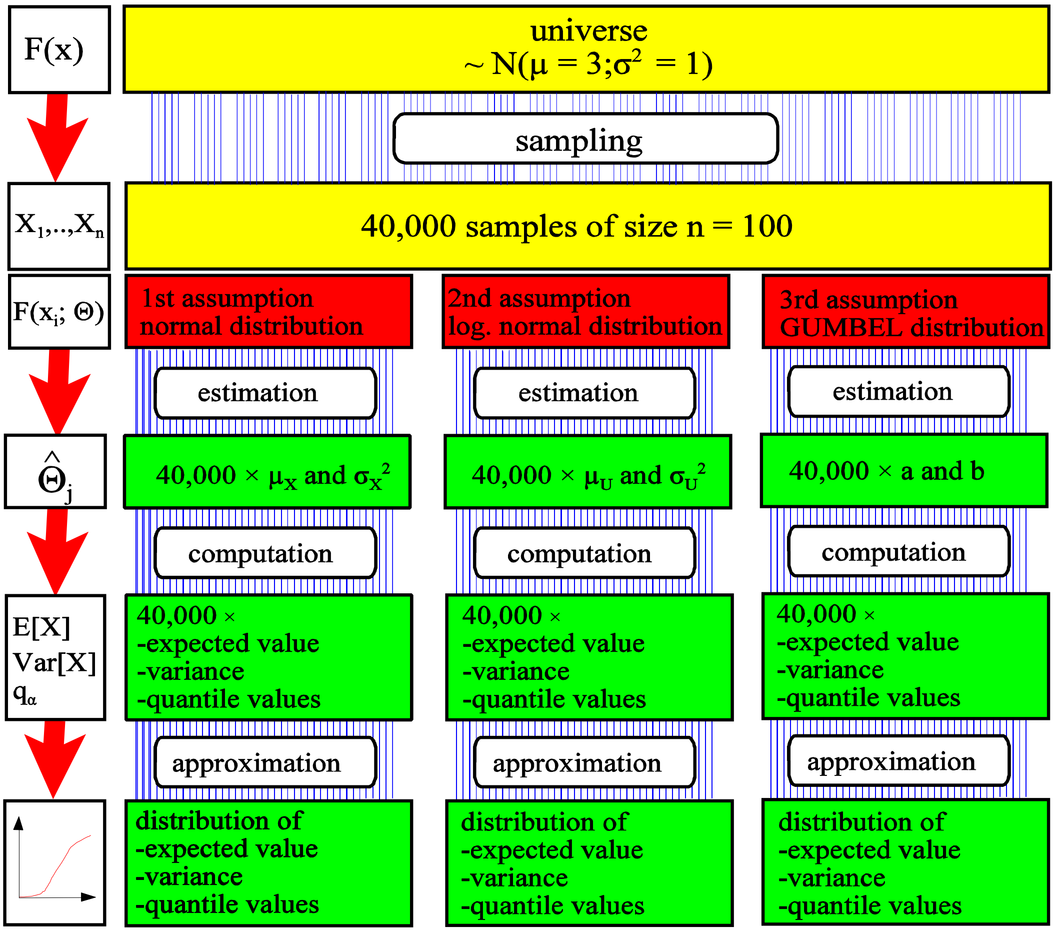


Figure 3: Schematic representation of the numerical experiment

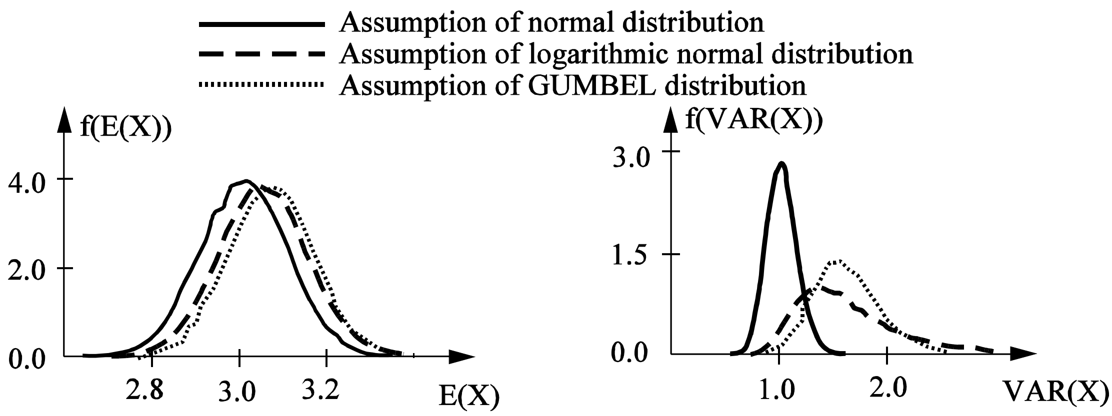


Figure 4: Distribution of the expected value E(X) and the variance VAR(X) assumed different types of probability distribution functions

In the case of non-constant reproduction conditions, a limited number of samples, uncertain or non-numerical data, or non-compliance with the i.i.d. paradigm, significant uncertainty exists, which must be described by extended uncertainty models. The modeling of uncertainty as fuzzy randomness leads to a generalized uncertainty model containing the special cases "fuzziness" and "randomness". The mathematical description of fuzzy randomness is based on the theory of fuzzy random variables. However, the fuzziness is modeled by means of fuzzy variables.

Modeling uncertain parameters as fuzzy variables is based on the fuzzy set theory, which is independent from statistical laws. The fuzzy set theory permits the consideration of subjective assessment by an expert in addition to objective physical parameters. Subjective influences may be numerically taken into account, processed and included in the evaluation of serviceability and structural safety. In contrast to probability density functions and probability distribution functions the required membership functions may be already specified on the basis of a few (or without) sample elements. Available information are assessed by means of membership functions considering expert knowledge. Linguistic assessment (e.g. the state of the building is structurally *very sound, sound, satisfying, suboptimal, bad*) may be included in the evaluation.

2.2 Fuzzy random variables

The underlying concepts and definitions relating to the theory of fuzzy random variables are expounded in [1], [2] and [3]. Fuzzy random variables are defined by an extension of the axiomatic probability concept after KOLMOGOROV. The probability space $[\underline{\mathbf{X}}; \mathfrak{S}; P]$ is thereby extended by the dimension of fuzziness; the uncertain measure probability remains defined over the n-dimensional EUCLIDian space $\mathbb{R}^n$.

A *fuzzy random variable* $\underline{\tilde{X}}$ is the fuzzy result of the uncertain mapping

$$\Omega \tilde{\rightarrow} \mathbf{F}(\mathbb{R}^n) \tag{5}$$

where $\mathbf{F}(\mathbb{R}^n)$ is the set of all fuzzy numbers in $\mathbb{R}^n$. Each real random variable $\underline{X}$ (without fuzziness) on $\underline{\mathbf{X}}$, which is completely contained in $\underline{\tilde{X}}$, is referred to as an original of $\underline{\tilde{X}}$. The fuzzy random variable $\underline{\tilde{X}}$ is the *fuzzy set of all possible originals* $\underline{X}$ contained in $\underline{\tilde{X}}$. The fuzzy random variable $\underline{\tilde{X}}$ may be described mathematically by a fuzzy probability distribution function $\tilde{F}(x)$.

The fuzzy probability distribution function $\tilde{F}(x)$ of $\underline{\tilde{X}}$ is the set of probability distribution functions of all originals $\underline{X}_j$ of $\underline{\tilde{X}}$ with the membership values $\mu(\tilde{F}(x))$, see Fig. 5. The quantification of fuzziness by fuzzy parameters leads to the description of the fuzzy probability distribution function $\tilde{F}(x)$ of $\underline{\tilde{X}}$ as a function of the fuzzy bunch parameter $\underline{\tilde{s}}$.

$$\tilde{F}(\underline{x}) = F(\underline{\tilde{s}}, \underline{x}) \tag{6}$$

For the purposes of numerical evaluation, α-discretization is advantageously applied.

$$F(\underline{\tilde{s}}, \underline{x}) = \{F_\alpha(\underline{x});\ \mu(F_\alpha(\underline{x}))\ |\ F_\alpha(\underline{x}) = [F_{min,\alpha}(\underline{x});\ F_{max,\alpha}(\underline{x})],\ \mu(F_\alpha(\underline{x})) = \alpha,\quad \forall \alpha \in (0,1]\} \tag{7}$$

with $F_{min,\alpha}(\underline{x}) = \inf\{F(\underline{s}, \underline{x})\ |\ \underline{s} \in \underline{s}_\alpha\}$

$F_{max,\alpha}(\underline{x}) = \sup\{F(\underline{s}, \underline{x})\ |\ \underline{s} \in \underline{s}_\alpha\}$

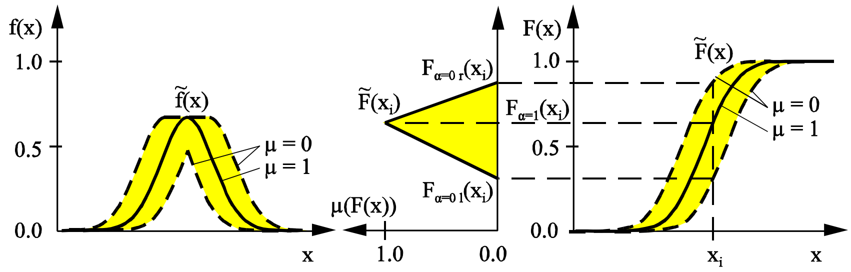


Figure 5: Fuzzy probability density function $\tilde{f}(x)$ and fuzzy probability distribution function $\tilde{F}(x)$ of a one-dimensional continuous fuzzy random variable

The generation of fuzzy probability distribution functions is explained below. These functions are formulated according to Eqn. (6) in relation to fuzzy bunch parameters. Fuzzy bunch parameters may constitute all functional parameters and/or parameters which describe the functional type.

The methods developed here for uncertain data analysis for the fuzzy evaluation of statistical inference are based on classical as well as modern statistics; bootstrap methods are also included [4]. Moment estimators and maximum likelihood estimators are applied for point and interval estimations of the distribution parameters. Assumed distribution types are assessed with the aid of goodness-of-fit tests. Non-parametric estimation methods for assessing samples (run test, test of homogeneity) are also applied. Two concepts were developed in order to take account of informal uncertainty. They were both applied in the example of ch. 4.1.

Fuzzy-Parameter estimation

Under the assumption of a functional type for the fuzzy probability distribution function, the estimation problem reduces to the determination of distribution parameters. These are modeled as fuzzy numbers. It is suggested here that the mean value of the fuzzy parameters is defined by point estimation and the α-level set by interval estimation for a prescribed confidence level. This approach is shown in Fig. 6 for the example of a fuzzy triangular number.

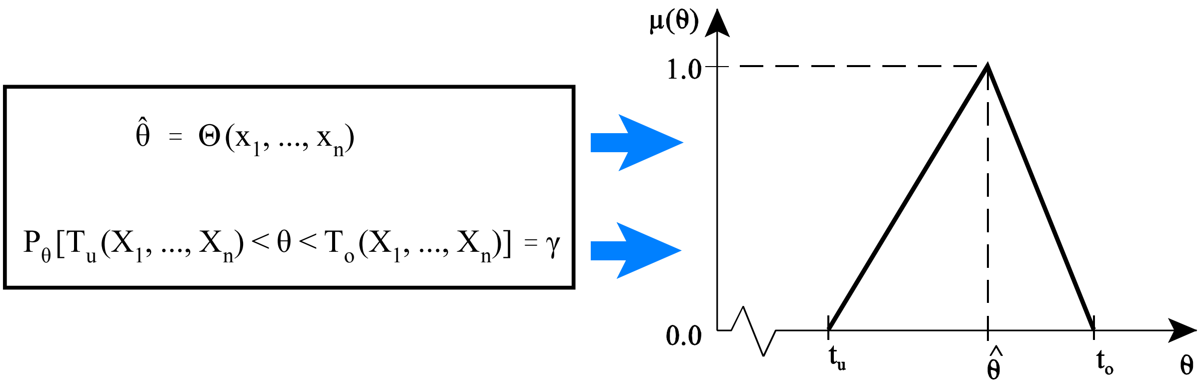


Figure 6: Determination of a fuzzy distribution parameter

Non-parametric estimation of the fuzzy probability distribution

This approach makes it possible to avoid the generally informal uncertain assumption of a distribution type insofar as the empirical distribution function is directly fuzzified and the fuzzy probability distribution function is directly constructed. The method is based on the principle that the empirical distribution function is comprised of binomially-distributed random variables. Applying the method after CLOPPER and PEARSON, confidence intervals for binomially-distributed random variables are determined [4]. These are used to fuzzify the functional values of the fuzzy probability distribution function.

3 Fuzzy probabilistic safety assessment

The aim of the fuzzy-probabilistic safety concept is to determine and assess the safety level of structures under consideration of fuzzy random variables. The result is the fuzzy failure probability or the fuzzy reliability index. The fuzziness of the computed safety level characterizes the new quality of the safety assessment compared with customary probabilistic methods.

The fuzzy failure probability is a measure indicating that the uncertain loading (external load) acting on the structure is larger than the uncertain loadability (resistance)

$$\tilde{P}_f = P\,(\,\tilde{R} - \tilde{S} \leq 0\,) \tag{8}$$

The loading and loadability are described by fuzzy random variables, random variables and fuzzy variables. Together with real random variables, the fuzzy random variables form the space of the basic variables X_i. Fuzzy variables are interpreted as model parameters of the uncertain structural model. In the space of the basic variables the survival region is separated from the failure region by the limit state function. According to the selected failure criterion it is possible to define different limit states (limit state of serviceability, limit state of load-bearing capacity). Uncertain computational models with fuzzy model parameters lead to a fuzzy limit state surface. The fuzzy model parameters are the bunch parameters of the fuzzy bunch of functions

$$\left(\tilde{g}(\underline{x}) = 0\right) = \left\{\left(g(\underline{x}) = 0; \mu(g(\underline{x}) = 0)\right) \mid \underline{x} \in \mathbf{X}\right\} \tag{9}$$

The fuzzy failure probability is obtained by integrating the fuzzy joint probability density function over the fuzzy failure region (Fig. 7)

$$\tilde{P}_f = \int\limits_{\underline{x} \mid \tilde{g}(\underline{x})<0} \cdots \int \tilde{f}_X(x_1, x_2, \ldots, x_n)\; dx_1\, dx_2 \ldots dx_n \tag{10}$$

With the aid of α-discretization the integral is evaluated on an original-to-original basis. The space of the bunch parameters of the fuzzy probability distribution functions is extended by the fuzzy model parameters. Realizations of the bunch parameters yield the joint density function $f_X(\underline{x})$ and the crisp limit state $g(\underline{x}) = 0$

$$P_f = \int\limits_{\underline{x} \mid g(\underline{x})<0} \cdots \int f_X(x_1, x_2, \ldots, x_n)\; dx_1\, dx_2 \ldots dx_n \tag{11}$$

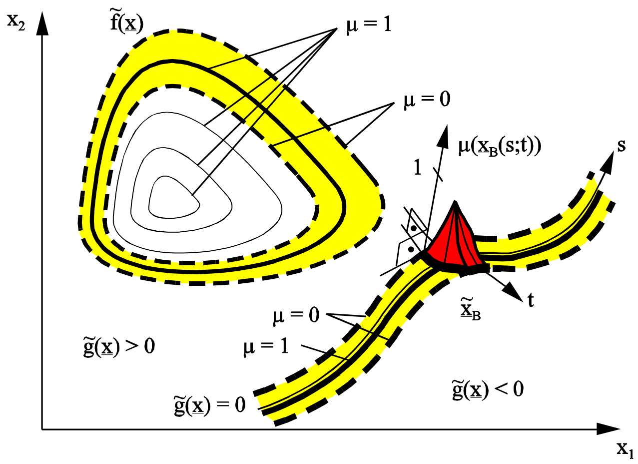


Figure 7: Fuzzy joint probability density function $\tilde{f}(\underline{x})$, fuzzy limit state surface $\tilde{g}(\underline{x}) = 0$ and fuzzy design point $\tilde{\underline{x}}_B$ with data and model uncertainty

The evaluation of the integral on an original-to-original basis, see Eqn. (11), is carried out using analytical methods, numerical integration, simulation methods and probabilistic approximation methods. The probabilistic approximation methods are further developed in such a way that it is possible to take account of basic variables in the form of fuzzy random variables as well as model parameters in the form of fuzzy variables. The first order reliability method (FORM) is extended to yield the first order fuzzy reliability method (FFORM) [5].

α-level optimization [6] is performed in the extended space of the fuzzy bunch parameters (s-space, fuzzy bunch parameters of the fuzzy probability distribution functions and fuzzy model parameters). The optimization targets are the smallest and largest failure probabilities on each α-level. The originals of the joint density function and the limit state function are known for discrete points in the s-space. The corresponding reliability index is determined using FORM. The result of the α-level optimization is the fuzzy reliability index $\tilde{\beta}$, which may be converted into the fuzzy failure probability $\tilde{P}_f$.

4 Examples

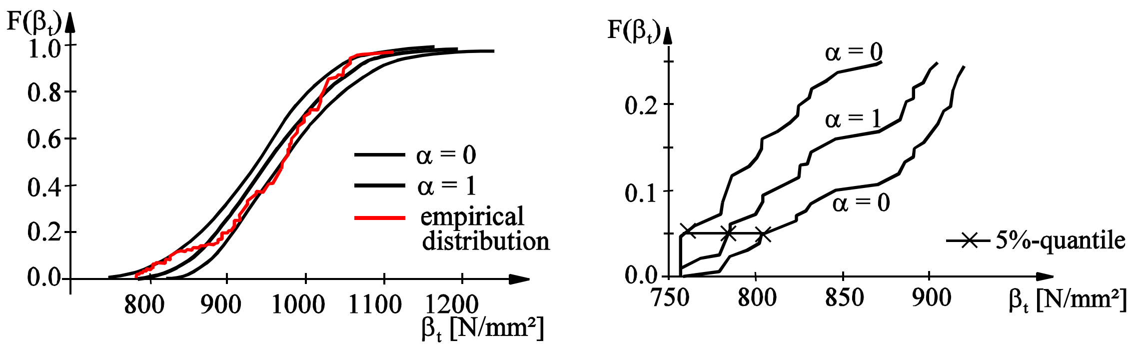
4.1 Tensile strength of textil reinforced test specimen - fuzzy probability distribution function

Among other things the tensile strength of textile reinforced strengthening layers is investigated with the aid of tensile tests of textile reinforced test specimen [7]. The breaking load is tested and divided by the cross-sectional area of the inserted textiles converted into the tensile strength,. Here the tensile strength of 120 test specimen, each with three reinforcement layers NWM3-019-00 respectively NWM3-028-00, is tested examplarily.

Non-parametric tests of the sample do not show any significant trends in respect of different test series. In order to determine the fuzzy probability distribution function normal distribution and logarithmic normal distribution are initially assumed for the distribution type. The parameters of the distribution functions are in each case estimated using the maximum likelihood method. Because of the small-sized sample the KOLMOGOROV-SMIRNOV goodness-of-fit test does not permit to reject one of the assumed distribution functions. Independent from the distribution type the mathematical description of tensile strength is objectively informally uncertain. Thus the parameters of the distribution are uncertain, because the probability P(X = x) that the parameters adopt certain values is zero. The uncertainty is appropriately taken into account, if the tensile strength are quantified by a fuzzy probability distribution function.

Fuzzy mean value und fuzzy variance are introduced as fuzzy triangular numbers. The support bounds ($\alpha = 0$) are determined by means of interval estimation with the two-side confidence level $\gamma = 0.95$. The result is the fuzzy normal distribution shown in Fig. 8a). The fuzzy probabilistic safety concept of ch. 3 is applied for the safety assessment.

The characteristic value of the tensile strength is determined by the 5% quantile of the fuzzy normal distribution with <781; 815; 845> [N/mm²] using a fuzzy triangular number. Because of the bad goodness-of-fit of the fuzzy normal distribution to the empirical distribution (s. Fig. 8a)) the fuzzy characteristic value is also determined by means of a non-parametric estimation (see ch. 2.2). The probability shadow of the empirical distribution is calculated by the two-side confidence level $\gamma = 0.95$. The fuzzy characteristic value is then the fuzzy triangular number <760; 775; 806> [N/mm²] (Fig. 8b)). It can be used for the determination of fuzzy partial safety factors.



a) Fuzzy parameter estimation with assumed normal distribution

b) Non-parametric estimation

Figure 8: Fuzzy probability distribution function of the tensile strength of textile reinforced strengthening layers

4.2 Fuzzy probabilistic safety assessment of a T-beam floor construction

The safety level for the limit state defined by system failure of the unstrengthened T-beam floor construction should be assessed with the aid of the fuzzy probabilistic safety concept of ch. 3 considering uncertainty of the physical parameters appropriately. This safety level supports the decision about structural strengthening. An increase of the loadability and safety level respectively is possibly adding a textile reinforced finegrade concrete layer. The expected increase of the safety level is to be demonstrated by a further investigation.

The T-beam floor construction is a part of an existing building. The load history consists of the dead weight and an only uncertain known live load p_1. The safety level concerning system failure is assessed with the aid of an incremental increase of the live load p_2 which is anticipated in the future. Exemplarily the live load p_2 is modeled by means of a fuzzy random variable. The live load p_1 and the concrete compressive strength are real random values. Fig. 10 shows the fuzzy probability density function $\tilde{f}(p_2)$. The bunch parameters of this function (expected value and standard deviation) are fuzzy triangular numbers (s. Fig. 10). The parameters of the GUMBEL distribution of live load p_1 are a = 2.565 and u = 5.699. The normal distribution of the concrete compressive strength is specified by the mean value m = 30 N/mm² and a variation coefficient v = 0.1.

The physical nonlinear deterministic fundamental solution is represented by the FE program FALT-FEM for the calculation of reinforced concrete folded plate structures. In this program the newly developed algorithms of project D2 (SFB 528) [8] are included concerning the nonlinear determination of structures with strengthening of preloaded systems. The supplementary addition of textil reinforced strengthening under consideration of the current stress and deformation state equals a modification of the cross section. The post-strengthening is modeled by rigid bond between the added layer and the old concrete.

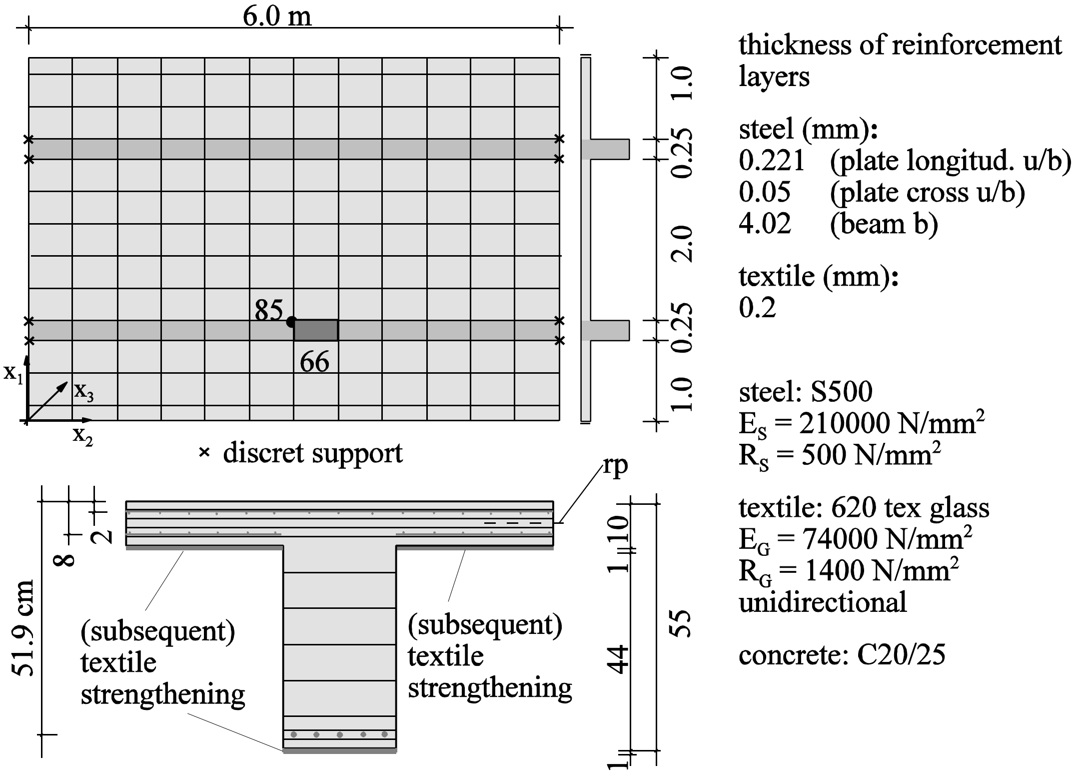


Figure 9: T-beam floor construction: geometry, FE mesh and material properties

The FFORM, see ch. 3, is used for fuzzy probabilistic safety assessment. The calculation results in the fuzzy reliability index $\tilde{\beta}$ shown in Fig. 11. The increase of the safety level due to strengthenig is clearly observable.

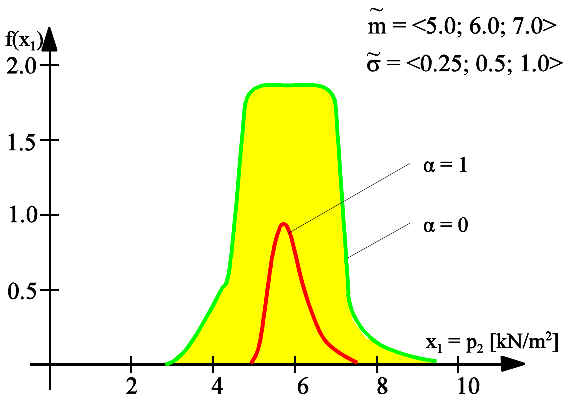


Figure 10: Live load p_2 - GUMBEL distribution

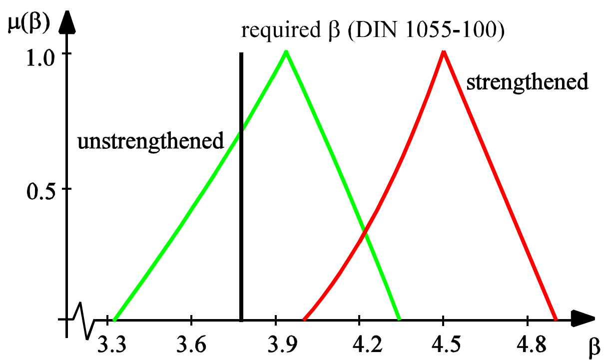


Figure 11: Fuzzy reliability index $\tilde{\beta}$

Exemplarily the load displacement dependencies of the unstrengthened and the strengthened structure are plotted and compared in Fig. 12 for certain parameters ($\mu = \alpha = 1$). Fig. 13 shows the stress development of the lower beam reinforcement of the finite element 66 during the load process. When the reinforcement steel attained its yield stress the textile stress began to arise strongly. The loadability and the structural safety of the T-beam floor construction may be increased because of the post-strengthening. The safety assessement without appropriate consideration of uncertainty of the input variables pretends a higher safety level ($\beta = 4.5$).

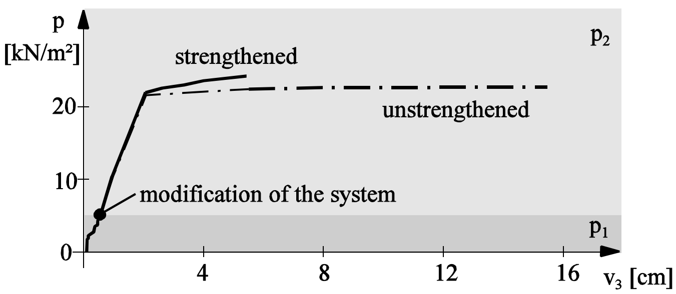


Figure 12: Load displacement dependency node 85

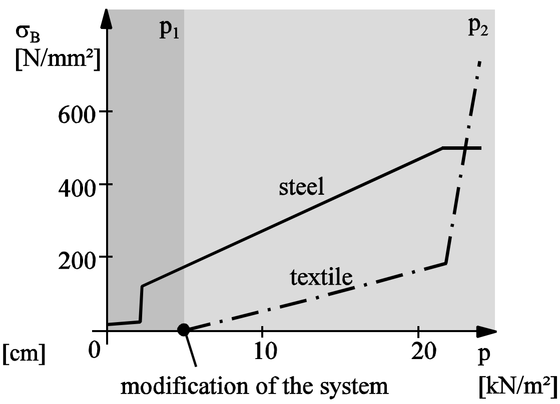


Figure 13: stress in the reinforcment

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References

[1] H. Kwakernaak, *Fuzzy random variables - I. Definitions and Theorems*, Information Sciences, 15, (1978), pp. 1-29, *Fuzzy random variables - II. Algorithms and Examples for the Discrete Case*, Information Sciences, 17, (1979), pp. 253-278.

[2] M.L. Puri, D. Ralescu, *Fuzzy random variables*, J. Math. Anal. Appl., 114, (1986), pp. 409-422.

[3] Liu Yubin, Qiao Zhong, Wang Guangyuan, *Fuzzy random reliability of structures based on fuzzy random variables*, Fuzzy Sets and Systems, 86, (1997), pp. 345-355.

[4] R. Schlittgen, *Statistische Inferenz*, Oldenbourg Verl., München, Wien, (1996).

[5] B. Möller, M. Beer, W. Graf, J.-U. Sickert, *Fuzzy probabilistic methods and its application for safety assessment of structures*. In: *2nd European Conf. on Computational Mechanics 2001*, Crakow, (2001), CD-ROM.

[6] B. Möller, W. Graf, M. Beer, *Fuzzy structural analysis using α-level-optimization*, Computational Mechanics, 26(6), (2000), pp. 547-565.

[7] F. Jesse, R. Ortlepp, M. Curbach, *Tensile Stress-Strain Behaviour of Textile Reinforced Concrete*. In: *Proceedings of the IABSE Symposium, "Towards a better built environment – innovation, sustainability, information technology"*, Melbourne, September 2002 by IABSE, ETH-Zurich, (www.iabse.ethz.ch) – CD-ROM and Book of Abstracts.

[8] W. Graf, B. Möller, A. Hoffmann, F. Steinigen, *Numerical simulation of RC structures with textile reinforcement*. In: *Fifth World Congress on Computational Mechanics*, Vienna, (2002), http://wccm.tuwien.ac.at