MODELING OF BLASTING PROCESSES IN VIEW OF FUZZY RANDOMNESS

B. Möller, A. Hoffmann, and M. Liebscher
Institute of Statics and Dynamics of Structures
Dresden University of Technology
Mommsenstr. 13, 01062 Dresden, Germany
moeller@rcs.urz.tu-dresden.de

Abstract

The objective of blasting operations is to shatter a building in such a way that the remaining debris can be removed easily. When blasting operations are carried out in urban areas, the surrounding properties should not be affected by debris or accompanying ground vibrations. To obtain a reliable prediction of the collapse process and its effects, a realistic simulation is necessary. In the stochastic sense, each blasting is an individual event that is characterized generally by limited data and distinctive data uncertainty. The mathematical description of data uncertainty is realized on the basis of *fuzzy randomness* (Möller and Beer, 2004). Fuzzy randomness is a generalized uncertainty model that includes fuzziness and randomness as special cases. In *fuzzy stochastic structural analysis* (Möller and Beer, 2004) the uncertain input parameters modeled by fuzzy randomness are mapped to fuzzy random results. The mapping is carried out by a so-called mapping model. In the case of blasting a multi body system algorithm is applied as mapping model. The structure is subdivided into rigid and flexible parts. Flexible parts represent potential failure zones that are subject to the major damages or destructions during the collapse of the structure. This mechanical behavior is modeled by nonlinear load-displacement-relations that describe crack development, articulation, and failure. The fuzzy stochastic analysis of blasting is demonstrated by way of an example.

1 Introduction

The controlled demolition of buildings that reached the end of their physical or economical lifetime is at present of increasing importance. On one hand, there is a large number of buildings that are derelict or which no longer meet the present requirements. On the other hand, the resources of building land are limited, in particular, in densely populated areas. Based on this situation, the demolition of existing structures often represents an economical and sometimes the only way to allow new building. For removing reinforced concrete structures, demolition by blasting has become a common practice for technical and economical reasons.

The collapse of the building is modeled with a multi body system algorithm. The structure is subdivided into rigid and flexible parts, see Figure 1. Flexible parts represent potential failure zones that are subject to the major damages or destructions during the collapse of the structure. They are modeled by 6 dof spring beam elements. The load-displacement behavior of the 6 dof spring beams elements is represented by 6 independent nonlinear load-displacement-relations. Contact situations between parts during the structure collapse are taken into account by the chosen simulation model.

Bringing down a building basically involves removing vertical supports – the columns and shear walls as highlighted in Figure 1 – in a controlled sequential way, that then as a result of the action of gravity the collapse of structure occurs. The basic idea is to weaken the structure on one side of the building's lower floors, starting at the bottom and working upward over a certain period of time. Each charge will cut through the concrete of a column and the weight of the structure above will induce the collapse. Shattered parts of the structure are removed in the computational simulation model to initiate the collapse process.

The goal of the simulations is a reliable prediction of blasting results in respect to uncertainty (randomness as special case included) of the mapping model's input parameters. Errors in the design of a blasting operation can be rather embarrassing, especially if the structure comes down where it isn't supposed to.

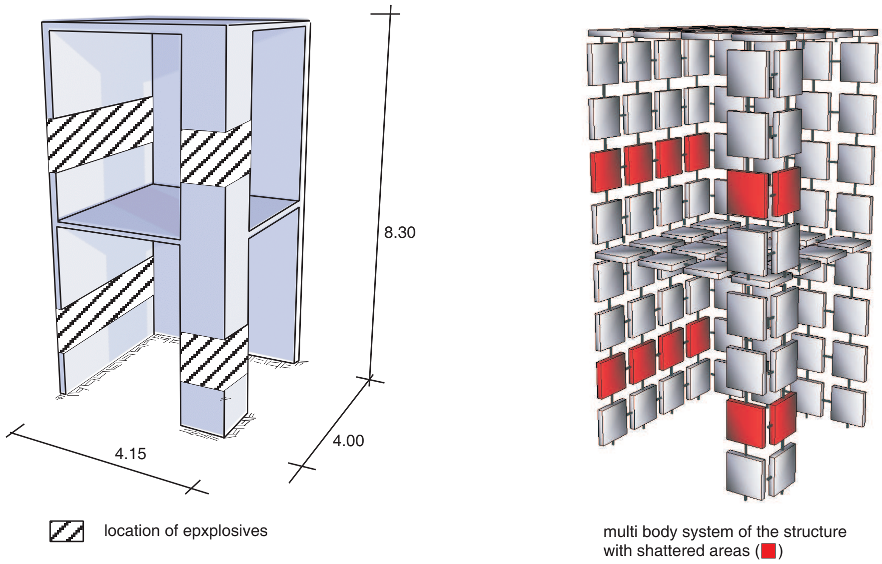

Figure 1: Multi body system of a simple two story building

2 Data Models Describing Uncertainty

Input parameters of a collapse simulation like material strength, the demolition effect of the explosives, and loadings are afflicted with uncertainty. In the stochastic sense, each blasting process is an individual event that is generally characterized by limited data. The classical modeling of these data using random variables is problematic. If a random event is uncertain then the uncertainty may be described by fuzzy randomness. Definitions and basic terms concerning fuzzy randomness have been introduced and enhanced in (Kwakernaak, 1978; Kwakernaak, 1979; Viertl, 1996). Based on α-level discretization and with the aid of the fuzzy probability distribution functions a representation of fuzzy randomness suitable for numerical simulations an basis of (Möller and Beer, 2004) is presented.

In the probability space $\mathbf{\Omega}$ a fuzzy realization of the form $\underline{\tilde{x}}(\omega) = (\tilde{x}_1, \ldots, \tilde{x}_n) \subseteq \underline{\mathbf{X}}$ is assigned to each event $\omega \in \mathbf{\Omega}$. The n-tuple $\underline{\tilde{x}}(\omega)$ is constituted from the n *fuzzy numbers* $\tilde{x}_1, \ldots, \tilde{x}_n$ on the fundamental set $\underline{\mathbf{X}} \in \mathbb{R}^n$. Each fuzzy number is defined as a convex, normalized fuzzy set

$$\tilde{x}_i = \{x, \mu_{x_i}(x) \mid x \in X\}\,. \qquad (1)$$

A *fuzzy random vector* $\underline{\tilde{X}}$ is the result of the uncertain mapping

$$\underline{\tilde{X}}: \quad \mathbf{\Omega} \;\; \tilde{\rightarrow} \;\; \mathbf{F}(\underline{\mathbf{X}})\,, \qquad (2)$$

in which $\mathbf{F}(\underline{\mathbf{X}})$ characterizes the set of all fuzzy numbers on $\mathbb{R}^n$.

The fuzzy probability distribution function $\tilde{F}(\underline{x})$ of the fuzzy random vectors $\underline{\tilde{X}}$ is defined as set of fuzzy probability distribution functions $F_j(\underline{x})$ of all originals $\underline{X}_j$ of $\underline{\tilde{X}}$ with the membership values $\mu(F_j(x))$. In bunch parameter representation with α-discretization applied $\tilde{F}(\underline{x})$ is defined by

$$\tilde{\underline{F}}(\underline{x}) = \underline{F}(\tilde{\underline{s}}, t) = \{(\underline{F}_\alpha(\underline{x});\ \mu(\underline{F}_\alpha(\underline{x}))\ |\quad \underline{F}_\alpha(\underline{x}) = [\underline{F}_{\min,\alpha}(\underline{x});\ \underline{F}_{\max,\alpha}(\underline{x})];\ \mu(\underline{F}_\alpha(\underline{x})) = \alpha,\ \forall\ \alpha \in (0,1]\}\,. \tag{3}$$

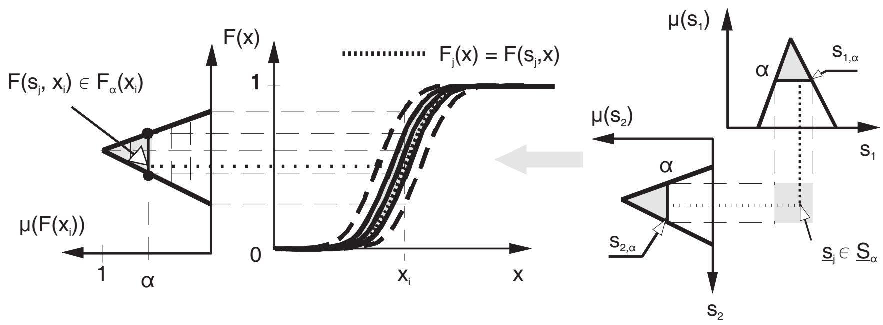

Figure 2: Fuzzy bunch parameter representation

A fuzzy random function $\tilde{\underline{X}}(\underline{t})$ is a function whose functional values are fuzzy random vectors. These functional values may depend on the spatial coordinates $\underline{\theta} = (\theta_1, \theta_2, \theta_3)$ and the time τ from the parameter space $\underline{\mathbf{T}} \subseteq \mathbb{R}^m$. With the crisp parameter vector $\underline{t} = (\tau, \theta)\ |\ \underline{t} \in \underline{\mathbf{T}}$ is the functional argument of a fuzzy random function defined on the space $\underline{\mathbf{T}} \times \Omega$. $\tilde{\underline{X}}(\underline{t})$ is defined as fuzzy result of the uncertain mapping

$$\tilde{\underline{X}}(\underline{t}):\quad \underline{\mathbf{T}} \times \Omega \ \ \tilde{\rightarrow}\ \ \mathbf{F}(\underline{\mathbf{X}})\,. \tag{4}$$

For each specified point $\underline{t} \in \underline{\mathbf{T}}$ a fuzzy random function represents a fuzzy random vector $\tilde{\underline{X}}_t$. A Fuzzy random function may also be defined as set of fuzzy random vectors on the parameter space $\underline{\mathbf{T}}$

$$\tilde{\underline{X}}(t) = \{\underline{X}_t = \tilde{\underline{X}}(\underline{t})\ \forall\ \underline{t}\ |\ \underline{t} \in \underline{\mathbf{T}}\}\,. \tag{5}$$

3 Data Uncertainty in Blasting Processes

To control the falling direction supports are taken out to force the building to fall in a specific direction. While this sounds simple, planning a successful drop is quite complicated. The 6 dof spring beam elements in the multi body system representing the stiffness of the structure play an important roll for a effective simulation. The load-displacement-relation of the 6 dof spring beam elements are computed by an FEM algorithm and depend e.g. on configuration of the reinforcements, concrete strength, and failure behavior. These parameters are characterized with uncertainty. The FEM algorithm describing the mapping operator is affected by fuzziness, randomness and fuzzy randomness of the input parameter. The result of the mapping is a fuzzy random load-displacement-relation. A fuzzy random load-displacement-relation $\tilde{F}(M(\phi))$ for $M(\phi)$ is examplarily shown in Figure 3.

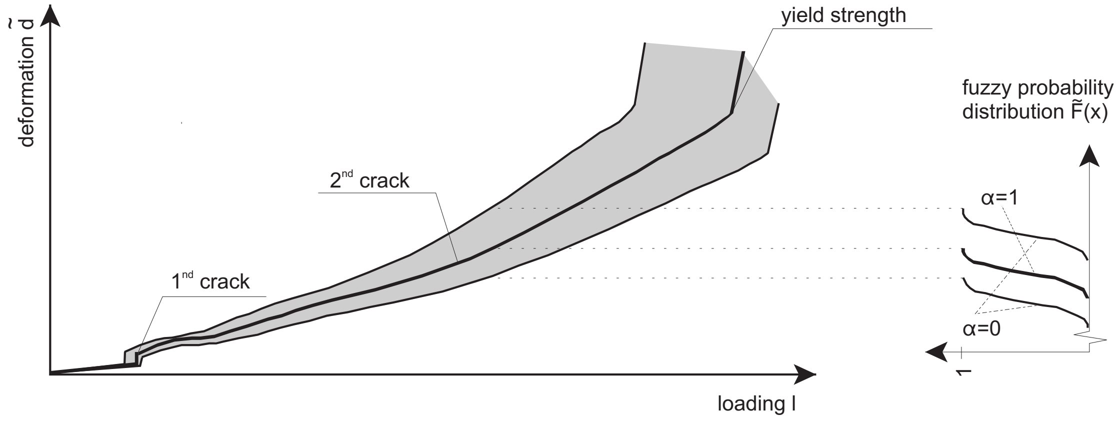

Figure 3: Fuzzy random force-displacement-relation

4 Fuzzy Stochastic Structural Analysis

4.1 General Procedure

If the uncertainty of the input parameters of a structural analysis is described with the aid of fuzzy random functions, the following problem is then to be solved for a crisp mapping model

$$F:\ \underline{\tilde{X}}(\underline{t}) \rightarrow \underline{\tilde{Z}}(\underline{t}). \tag{6}$$

Applying Eq. (6) the fuzzy random functions (structural input parameters) $\underline{\tilde{X}}(t)$ are mapped onto the fuzzy random functions (structural responses) $\underline{\tilde{Z}}(t)$. As fuzzy vectors and real random vectors are special cases of fuzzy random functions, these uncertainty models are also accounted for with Eq. (6). The mapping according to Eq. (6) is the symbolic representation of a fuzzy stochastic structural analysis, that is here a fuzzy stochastic multi body dynamic problem.

A multi body dynamic algorithm may be described on the basis of the nonlinear differential equations

$$\tilde{m} \cdot \underline{\ddot{\tilde{r}}} = \sum_i \underline{\tilde{F}}_i\,, \tag{7}$$

$$\tilde{J}_s \cdot \underline{\dot{\tilde{\omega}}}_s = \underline{\widetilde{M}}_s + \sum_i (\underline{\tilde{r}}_i \times \underline{\tilde{F}}_i)\,. \tag{8}$$

4.2 Numerical Realization

It is now intended to compute the structural responses $\underline{\tilde{Z}}(\underline{t})$ according to Eqs. (7) and (8) as fuzzy random vectors $\underline{\tilde{Z}}_{t_r} = \underline{\tilde{Z}}(\underline{t}_r)\ \mid\ r = 1,\ldots,q_1$ with the fuzzy probability distribution functions $\tilde{F}_{t_r}(\underline{z}) = \tilde{F}(z,\underline{t}_r)$ $= F(\underline{\tilde{\sigma}}_r, \underline{z}, t_r)$ at q_1 points $\underline{t}_r$ in the parameter space $\mathbf{T}$. For this purpose q_1 fuzzy bunch parameter vectors $\underline{\tilde{\sigma}}_r$ are to be determined, which comprise a total of m_1 bunch parameters $\tilde{\sigma}_1,\ldots,\tilde{\sigma}_{m_1}$. The m_1 fuzzy bunch parameters are combined in the fuzzy vector $\underline{\tilde{\sigma}}$. The fuzzy stochastic structural analysis characterized by Eq. (6) has thus been transformed into the mapping

$$F:\ \underline{\tilde{s}} \rightarrow \underline{\tilde{\sigma}}. \tag{9}$$

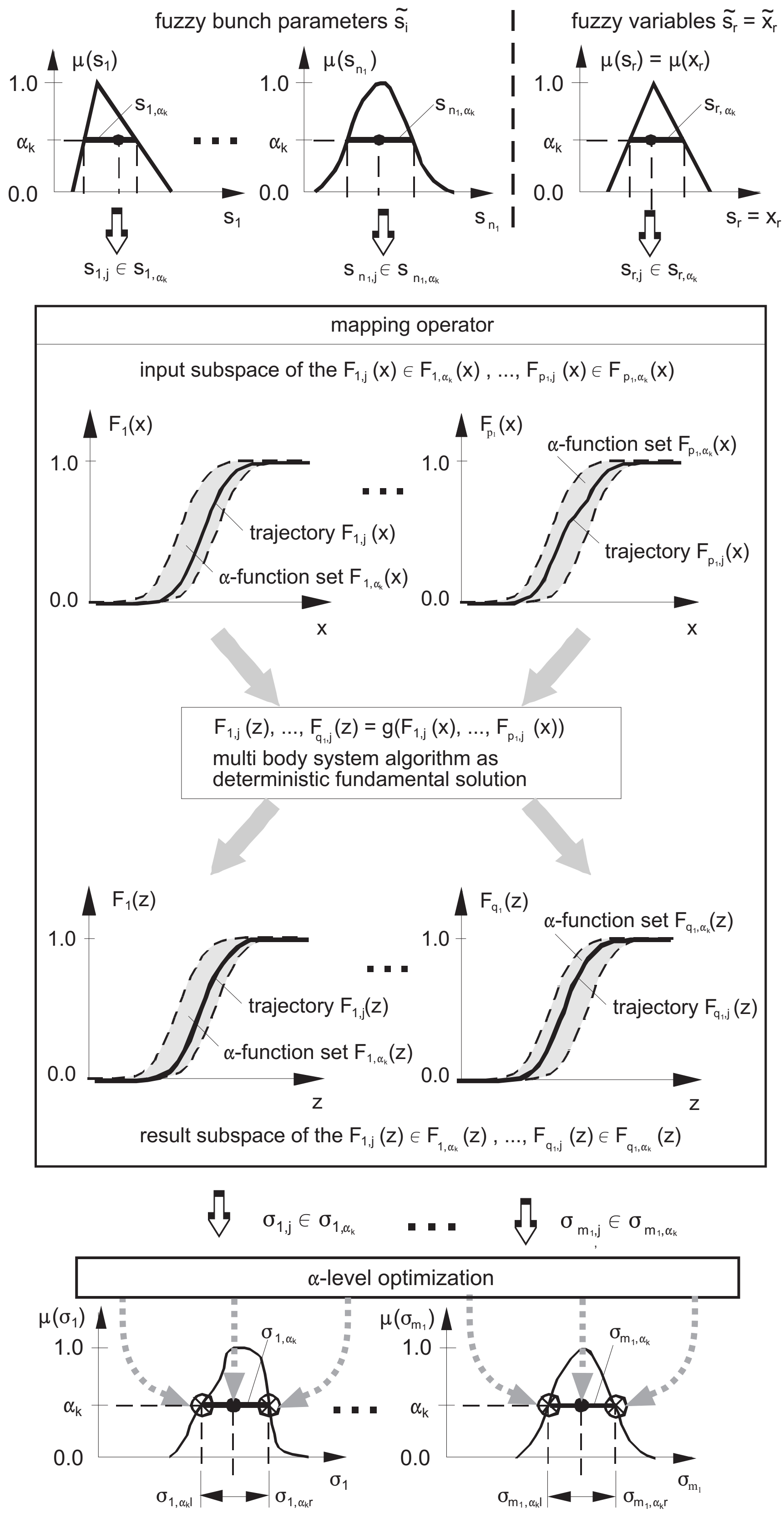

Figure 4: Numerical Realization of the fuzzy stochastic analysis

An α-discretization of the fuzzy bunch parameters belonging to $\tilde{s}_1, \ldots, \tilde{s}_{n_1}$ fuzzy probability distribution functions $F(\underline{\tilde{s}}_i, \underline{x}, t_r)$ yields the α-level sets $S_{1,\alpha_k}, \ldots, S_{n_1,\alpha_k}$ for the level α_k (Figure 4). These α-level sets together with the α-level sets $S_{h,\alpha_k} \mid h = n_1 + 1, \ldots, n$ form the n-dimensional crisp subspace $\underline{S}_{\alpha_k}$. If one element is selected from each α-level set, one crisp point s is then obtained in the subspace $\underline{S}_{\alpha_k}$.

With each set of crisp elements $s_{1,j} \in S_{1,\alpha_k}, \ldots, s_{n_1,j} \in S_{n_1,\alpha_k}$ constituting the vector $\underline{s}_j \in S_{\alpha,k}$ precisely one trajectory $F_{t_i,j}(\underline{x}) \in F_{t_i,\alpha_k}(x) \mid i = 1, \ldots, p_1$ with the membership value $\mu(F_{t_i,j}(\underline{x})) = \mu(s_j) \geq \alpha_k$ is simultaneously selected from each of the p_1 fuzzy probability distribution functions (Figure 4). The trajectories are $F_{t_i,j}(\underline{x})$ real-valued probability distribution functions. Each α-function set $F_{t_i,\alpha_k}(\underline{x})$ comprises all trajectories of the fuzzy probability distribution function $\tilde{F}_{t_i}(\underline{x})$ at the point $\underline{t}_i \in \underline{T}$ for the level α_k.

Having selected one crisp point s_j from the subspace $\underline{S}_{\alpha_k}$ one real probability distribution function (trajectory) is known for each fuzzy random vector $\underline{\tilde{X}}_{t_i} = \underline{\tilde{X}}(t_i)$. Moreover, one element from the respective α-level set (for the same level α_k) of each bunch parameter belonging to the fuzzy vectors, fuzzy fields, real random vectors, and fuzzy covariances is to be selected. Based on a multi body system algorithm one stochastic structural analysis may now be carried out for the crisp bunch parameter vector $\underline{s}_j \in \underline{S}_{\alpha_k}$ defined.

The trajectories of the fuzzy probability distribution functions $F(\underline{\tilde{\sigma}}, \underline{z}, \underline{t}_r)$ of the result vectors $\underline{\tilde{Z}}_{t_r} = \underline{\tilde{Z}}(\underline{t_r})$ $\mid r = 1, \ldots, q_1$ are designated by $F_{t_i,j}(\underline{z}) \mid r = 1, \ldots, q_1$. For each defined α-level α_k these are elements of the assigned α-function sets $F_{t_i,j}(\underline{z}) \in F_{t_i,\alpha_k}(\underline{z})$. For determining the α-function sets $F_{t_i,\alpha_k}(\underline{z})$ the following functional relationship concerning the trajectories may then be stated:

$$(F_{t_i,j}(\underline{z}) \mid r = 1, \ldots, p_1) = g(F_{t_i,j}(\underline{x}) \mid r = 1, \ldots, p_1) \qquad (10)$$

The solution of Eq. (10) is obtained with the aid of the Monte Carlo Simulation (MCS), see (Sickert et al., 2003). Based on the trajectories $F_{t_i,j}(\underline{x})$ sample vectors are thereby generated one after another. Each sample vector comprises exactly one realization of each original $\underline{X}_{t_i,j}$, respectively, and thus represents a crisp input vector for one deterministic structural analysis. The application of MCS results in a sample of result values for each trajectory $F_{t_i,j}(\underline{z})$ of the fuzzy random result vectors $\underline{\tilde{Z}}_{t_r} = \underline{\tilde{Z}}(\underline{t}_r)$. Statistical evaluation of these samples yields the trajectories in bunch parameter representation. For each α-level α_k the obtained bunch parameters are elements of the assigned α-level sets $\sigma_{1,j} \in \sigma_{1,\alpha_k}, \ldots, \sigma_{m,j} \in \sigma_{m_1,\alpha_k}$ of the fuzzy bunch parameters $\tilde{\sigma}_1, \ldots, \tilde{\sigma}_{m_1}$ constituting the fuzzy vector $\underline{\tilde{\sigma}}$. Once the smallest and largest elements of the α-level sets $\sigma_{1,\alpha_k}, \ldots, \sigma_{m_1,\alpha_k}$ have been determined for each α-level α_k, the fuzzy bunch parameter vectors $\tilde{\sigma}_1, \ldots, \tilde{\sigma}_{q_1}$ and hence the fuzzy probability distribution functions $\tilde{F}(\underline{z}, \underline{t}_r) = F(\underline{\tilde{\sigma}}, \underline{z}, \underline{t}_r)$ are then known. The search for the smallest and largest elements of the bunch parameters is realized by applying α-level optimization (Figure 4).

5 Example

The blasting simulation of the multi body system as shown in Figure 1 is presented. The stiffness represented by force-displacement-relations (Figure 3) is considered to be logarithmic normal distributed with the fuzzy expected value $\tilde{s}$ (Figure 5). The surrounding property must not be affected due to the blasting operation – this is fulfilled if the dashed border in Figure 6 is not exceeded by the debris. A fuzzy failure probability for exceeding the border by given fuzzy stochastic input parameters has to be determined. Results are shown in Figure 6.

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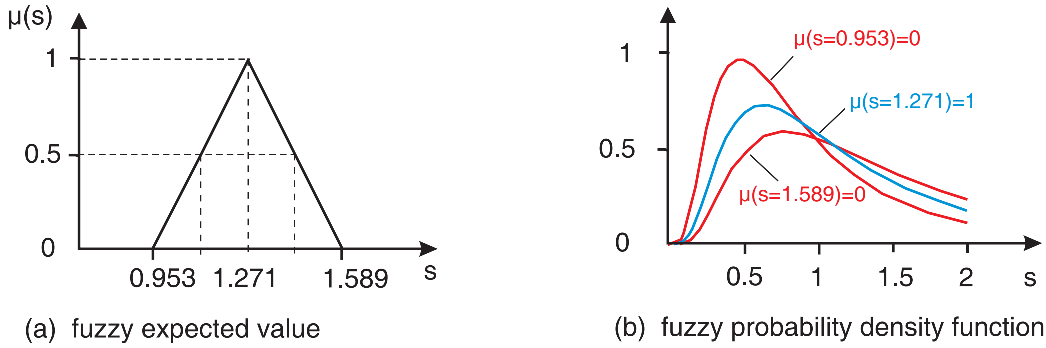

Figure 5: Fuzzy expected value and fuzzy probability density function

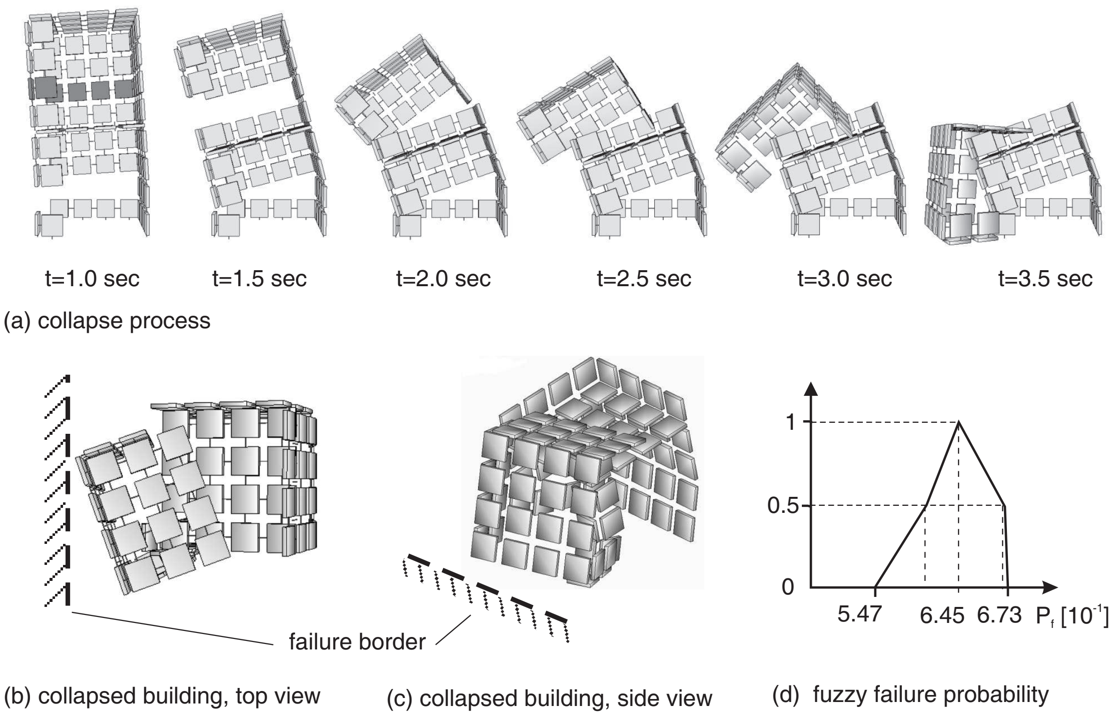

Figure 6: Result of collapse process simulation

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