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Fuzzy Cluster Design: A New Way for Structural Design

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Abstract

In this paper a fuzzy method for determining appropriate structural design is presented. Non-stochastic uncertainty is quantified using fuzzy values and fuzzy random variables. Fuzzy structural parameters are processed on the basis of a generally applicable numerical method for arbitrary linear and nonlinear fuzzy structural analyses. Fuzzy randomness is introduced into a fuzzy probabilistic safety assessment, which is formulated by extending common probabilistic concepts. From these applications fuzzy results are obtained as fuzzy structural responses and fuzzy safety levels. These fuzzy results are compared to permissible values and assessed using Shannon's entropy and defuzzification algorithms. Referring to permissible structural responses and safety levels uncertain structural design parameters are derived by applying a fuzzy cluster analysis to the fuzzy structural parameters and fuzzy parameters of fuzzy random variables. The nonlinear fuzzy structural analysis including uncertain structural design is demonstrated by way of an example.

Keywords: fuzzy numbers, fuzzy random variables, fuzzy structural analysis, fuzzy probabilistic safety assessment, fuzzy clustering, uncertain structural design, nonlinear structural design.

1 Uncertainty in structural engineering

Structural engineering mainly focuses on computing structural responses, assessing structural safety, and determining parameters for structural design that meets all relevant requirements. For these purposes, the structural engineer has to apply appropriate structural models, suitably-matched computational models and reliable structural parameters close to reality. Computational models must be capable to numerically simulate the system behavior of the chosen structural model. They have already been developed up to a high quality level and are available as nonlinear numerical procedures for solving many problems. Structural models and structural parameters, however, have to be established in the particular case on the basis of plans, drawings, measurements, observations, experiences, expert knowledge, codes and standards. As a rule certain information regarding structural models and precise values of structural parameters do not exist. Structural models and structural parameters are significantly characterized by uncertainty. Human mistakes and errors in the manufacture, the use and maintenance of constructions, expert evaluations, and insufficient information sources represent only some examples of uncertain influences. In order to perform realistic structural analysis and safety assessment this uncertainty must be appropriately taken into consideration.

Different methods are available for mathematically describing and quantifying uncertainty. Some of these basic concepts are e.g. probability theory [1], including subjective probability approach [2] and Bayes methods [3], interval mathematics [4], convex modeling [5], theory of rough sets [6], fuzzy set theory [7], theory of fuzzy random variables [8] and chaos theory [9]. In the scientific literature the new uncertainty models are not only controversially discussed [10] but also increasingly implemented for the solution of practice-relevant problems [11–18]. These different developments of uncertainty models do not directly contradict each other but rather constitute an entirety.

The procedure presented in this paper exclusively takes account of uncertainty that may be quantified using fuzzy values and fuzzy random variables. Fuzzy structural parameters are mapped onto fuzzy result values by means of the fuzzy structural analysis. For this purpose a generally applicable as well as efficient numerical algorithm has been developed and formulated in terms of α-level optimization [19]. Fuzzy random variables are quantified by fuzzy probability distributions and introduced into the fuzzy probabilistic safety assessment. Based on the developed Fuzzy First Order Reliability Method (FFORM) [20] fuzzy safety prognoses are derived. The algorithms for processing uncertainty are linked to deterministic fundamental solutions that must be able to describe the load-bearing behavior of the structure in a sufficiently realistic manner. Thus a computational model that is capable to take account of all essential nonlinearities is applied.

On the basis of the results from fuzzy structural analysis and fuzzy probabilistic safety assessment an appropriate structural design has to be derived. Due to the uncertainty of structural parameters an uncertain structural design fits the reality in the best way. That means that modified fuzzy structural parameters and fuzzy parameters of fuzzy random variables are determined as being uncertain design values. By comparing requirements regarding structural responses and safety levels with the fuzzy results, permissible points in the space of the fuzzy input values are detected. Permissible uncertain design values are derived on the basis of a fuzzy cluster analysis algorithm in the space of the fuzzy input values.

2 Processing uncertainty in structural analysis and safety assessment

For introducing uncertain structural parameters into structural analysis and safety assessment these must be quantified. Depending on the characteristic of the uncertainty different mathematical basics are applied. For assigning appropriate mathematical models the uncertain parameters are classified according to the cause of their uncertainty. If exclusively informal or lexical uncertainty appears, the uncertainty characteristic *fuzziness* may be stated. If the uncertain parameter considered is partly influenced by stochastic uncertainty, but cannot be described using random variables without an element of doubt, then the characteristic *fuzzy randomness* may be assigned. Thereby real valued randomness may be treated as a special case of fuzzy randomness, these represent uncertain variables whose uncertainty exclusively arises from stochastic causes.

Uncertain parameters whose uncertainty characteristic has been identified as *fuzziness* are treated on the basis of the fuzzy set theory. They are described as uncertain sets $\tilde{A}$ and quantified by membership functions $\mu(\mathrm{x})$. The result of this fuzzification is the fuzzy value $\tilde{\mathrm{x}}$.

In order to specify the membership function of an uncertain set it is not possible to state a general algorithm [21]. The only requirement is compliance with the conditions for membership functions in accordance with the mathematical basics.

When fuzzifying the uncertain structural parameters a distinction between data and model uncertainty is made. Fuzzy model parameters are interpreted as being fuzzy values that are not chosen to be fuzzy design values but are nevertheless characterized by uncertainty. They are fuzzified in the common way, i.e. they should only mirror real uncertainty without any understatement or exaggeration in the scale of uncertainty. On the other hand, fuzzy design parameters, i.e. the fuzzy input values of the uncertain structural design process, have to be defined in a more comprehensive way. These uncertain parameters are to be fuzzified on a grand uncertainty scale. Their uncertainty should cover the whole parameter range imaginable for the design and include preferable parameter values weighted by higher membership values than the other ones. The fuzzified design parameters serve as an envelope, included in which the final uncertain structural design will be determined.

All fuzzified structural parameters are together introduced into fuzzy structural analysis. Fuzzy structural analysis implies the analysis of a structure with the aid of a crisp (or uncertain) algorithm applied to fuzzy values for input and model parameters. In fuzzy structural analysis the deterministic algorithms for statical and dynamic computations are adopted as a deterministic fundamental solution. The fuzziness of uncertain input and model parameters is processed on the basis of the developed α-level optimization [19]. The solution technique is formulated in terms of a modified evolution strategy that targets at a minimal computational effort. This concept permits to implement an arbitrary nonlinear deterministic fundamental solution without any special properties.

The developed algorithms of fuzzy structural analysis yield sets of discrete points in the space of the fuzzy result values. These points are evaluated by membership values. Additionally, the parameter coordinates in the space of the fuzzy input values and fuzzy model parameters which belong to these result points are known.

Uncertainty with the characteristic *fuzzy randomness* is described, quantified, and processed on the basis of the theory of fuzzy random variables. For quantifying fuzzy random variables fuzzy probability distributions are introduced [21, 22]. These may be understood as being a bunch of real valued probability distributions assessed by membership values μ indicating their degree of plausibility. The fuzzy probability distribution functions are analytically described by introducing fuzzy functional parameters into the common equations for distribution functions. Additionally fuzzy functional types of distributions may be defined. The fuzzy parameters of the fuzzy probability distributions represent fuzzy values characterized by membership functions as described above. They may also be divided into fuzzy model parameters and fuzzy design parameters with consequence to their fuzzification. When fuzzifying these parameters common methods of statistical mathematics may be applied [22]. For example, interval estimations may be used as orientation aids for defining intervals of parameters on different membership levels.

The quantified fuzzy random variables are introduced into the fuzzy probabilistic safety assessment. The method that has been developed for that purpose represents a comprehensive safety concept, which is formulated as a further development of introduced probabilistic approaches. This further development is described by way of the Fuzzy First Order Reliability Method (FFORM) in [20].

The aim of this fuzzy probabilistic safety assessment is to determine and assess the fuzzy safety level. Fuzzy random variables, real random variables, and fuzzy variables may thereby be accounted for simultaneously. The uncertainty of the input data and of the (computational) model is apparent in the results of the safety assessment, i.e. in the fuzzy failure probability and the fuzzy reliability index. These fuzzy results are obtained in the same form as the fuzzy structural responses, i.e. as discrete point sets.

In the following fuzzy structural parameters and fuzzy parameters of fuzzy random variables which are chosen to be fuzzy design values are together named fuzzy input values.

3 Determination of permissible fuzzy design values

The developed concept for uncertain structural design permits to design a structure on the basis of an arbitrary nonlinear structural analysis and additionally takes account of uncertainty. It makes use of the results from fuzzy structural analysis and fuzzy probabilistic safety assessment. For the first time this concept provides a capable tool for determining permissible uncertain structural parameters and for assessing fuzzy input and fuzzy result values.

The problem to be solved is that nonlinear fuzzy structural analysis and fuzzy probabilistic safety assessment do not permit to conclude directly from fuzzy results back to the assigned fuzzy input values. There are only few quasi arbitrary distributed crisp points in the space of the fuzzy input values, for which assigned crisp result values are known.

Attempt is made to derive permissible fuzzy design parameters by clustering the known crisp points in the space of the fuzzy input values. According to requirements concerning serviceability and load bearing capacity limit values for structural responses and safety levels are defined and compared to the fuzzy results from fuzzy structural analysis and fuzzy probabilistic safety assessment, see Fig. 1.

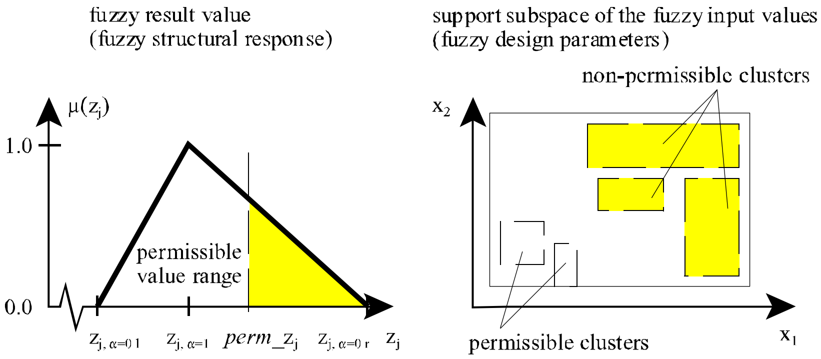

Figure 1: Assessing fuzzy structural response and clustering permissible points in the input space

In the case that all fuzzy results fulfill the requirements completely, structural design parameters may be chosen arbitrarily within the uncertainty of the fuzzified input parameters. If already one of the fuzzy results does not even include one permissible point, no structural design can be derived within the considered uncertainty. Otherwise, i.e. all fuzzy results are at least partly permissible but some of them are not completely permissible, crisp permissible and crisp non-permissible points may be determined in the space of the fuzzy input values. Uncertain structural design values can then be determined in the following three steps:

1. Starting from the detected crisp permissible input points a conclusion is drawn to determine permissible domains inside the uncertain input set $\underline{\tilde{X}}$. For that purpose cluster analysis algorithms offer a suitable basis. Using a fuzzy cluster algorithm several alternative sets of modified fuzzy input parameters representing permissible fuzzy design values may be generated from permissible point crowds inside $\underline{\tilde{X}}$.

2. The generated sets of modified fuzzy input values one after the other are again introduced into fuzzy structural analysis and fuzzy probabilistic safety assessment. As only permissible clusters are chosen for generating modified fuzzy

input values the assigned fuzzy results are expected as being permissible to a high probability. Nevertheless, the fuzzy results must be compared to the given requirements again. If a fuzzy result includes non-permissible values the assigned set of fuzzy input values must be modified. This may be done either by repeating the cluster analysis (step 1) using modified parameters or by clustering permissible points only inside the modified uncertain input set of this particular set of fuzzy input values.

3. After having determined different sets of fuzzy input values that meet all requirements these alternative sets of permissible fuzzy design values are compared to each other. The assigned results are assessed according to both their value and uncertainty. Whereas the value of the fuzzy results informs about the degree of the compliance with the requirements their uncertainty measures the structural sensibility. From this assessment the "best" set of permissible fuzzy design values is chosen as being the basis for the construction. As not only a permissible but also a robust structural design is determined the new concept for uncertain structural design may be understood as being a "two-dimensional design concept".

In the following the main parts of the new concept are discussed more in detail.

3.1 Cluster analysis and generation of modified fuzzy input values

All points in the uncertain input set $\underline{\tilde{X}}$ that are processed in fuzzy structural analysis and fuzzy probabilistic safety assessment are crisply separated into permissible and non-permissible points representing two subsets of $\underline{\tilde{X}}$. These two subsets are separately introduced into cluster analysis whereby the membership values of the points are not taken into account. Thereby each point represents one element in the sense of cluster analysis. Cluster analysis permits to classify elements (objects) that are defined by a character vector. Here the character vector is established by the coordinates of the input points. Elements that possess similar character vectors are to be assigned to the same cluster whereas elements that possess different character vectors are rather to be assigned to different clusters.

The clustering targets at preferably large clusters and a clear demarcation between permissible and non-permissible clusters. For that purpose several cluster analysis methods are available, see e.g. [23–25]. Numerical investigations and applications to representative examples indicated the fuzzy cluster algorithm after [24] as being the most suitable one. Fuzzy clustering possesses an essential advantage in contrast to crisp cluster methods; the elements are not uniquely assigned to one cluster but get a membership value μ characterizing the degree of their assignment to several clusters.

When applying fuzzy cluster analysis to the points in the uncertain input set $\underline{\tilde{X}}$ two important aspects must be considered.

3.2 Determining a suitable number of clusters

The number k of clusters may not be determined in advance, but the quality of clustering depends on this number. Therefore alternative partitionings are investigated by prescribing a value range $\mathrm{k} = 2, \ldots, \mathrm{n}$ and varying the number k. The results for every value k are assessed using quality indexes, which permit to determine a suitable partitioning according to an advantageous number of clusters.

For assessing clustering quality several quality indexes, which focus on different aspects, are available. The use of different indexes leads to different ratings of possible numbers k. Thus as a rule a multiple solution is obtained. Depending on the particular problem concerned the most suitable number of clusters has to be chosen in the sense of a compromise solution. The most favorable results have been gained using the quality indexes "separation degree" [23] and "normalized partition coefficient" [25]. In the case that only two fuzzy input values are introduced in cluster analysis the search for the best partitioning may be supported by a graphical representation.

Furthermore, the overlapping of permissible and non-permissible clusters is to be minimized. Though the number of overlapping zones can be determined for a given number k, the mode of overlapping can hardly be quantified. Two basic approaches are proposed to solve this problem:

1. The permissible cluster is to be divided into non-overlapping subclusters.
2. For a given number k the partitioning may be modified by prescribing minimum membership values μ for assigning elements to clusters. The higher the value is required the smaller the overlapping zones are obtained.

3.3 Choosing a suitable α-level

The information contained in the membership values of the elements can be exploited to generate a suitable partitioning. Clusters obtained on the α-level α= 0 often cover the whole support of the uncertain input set $\underline{\tilde{X}}$ and thus cause overlapping of permissible and non-permissible clusters. This indicates the necessity of prescribing a minimum cluster membership of elements. Stepwise increasing of the α-level leads to cluster kernels, which are non-overlapping and thus represent a suitable basis for generating permissible fuzzy design values, see Fig. 2.

One after the other the permissible clusters may be directly used to generate modified fuzzy input values. This means that each cluster leads to one set of modified fuzzy input values. In the case that many small permissible clusters exist, this way yields less suitable results. As larger clusters are preferable they may be constructed artificially by merging small clusters to superclusters.

To generate modified fuzzy input values at least their supports and mean values $(\mathrm{x} \mid \mu(\mathrm{x}) = 1)$ must be determined. The supports are given by the borders of the largest permissible clusters that do not overlap to non-permissible ones. Mean values and additional clues can be gained as follows:

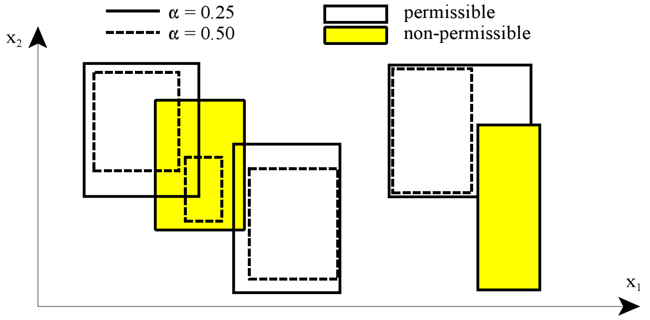

Figure 2: Determination of non-overlapping clusters by increasing the α-level

1. The cluster center may serve as mean value.

2. The mean value may be determined by the representative cluster element (prototype), see Fig. 3.

3. If the mean value from the prior fuzzification lies within the modified support, this may be maintained. This value has initially been evaluated as being preferable and thus leads to a reasonable mean value for the modified fuzzy input value.

4. Additional information may be gained from clustering on different α-levels. The assigned cluster bounds may serve as orientation aids for the α-level sets of the modified fuzzy input values, see Fig. 3. This method provides the best adaption to the cluster analysis results.

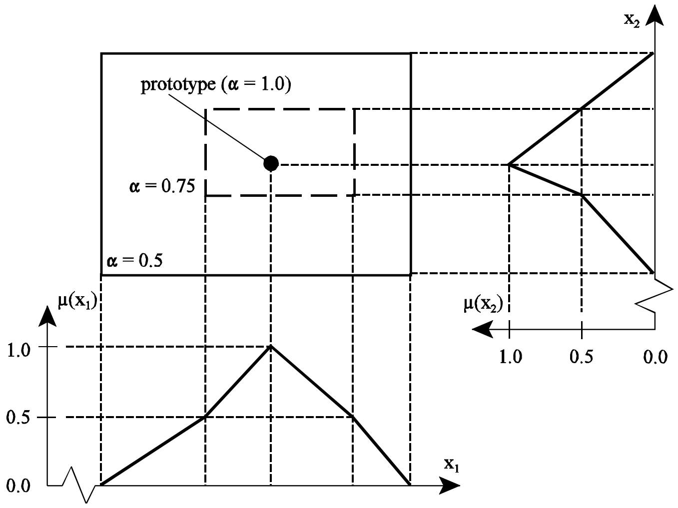

Figure 3: Generation of modified fuzzy input values from fuzzy cluster analysis results

3.4 Assessment of the modified fuzzy results

The generated sets of modified fuzzy input values represent alternative structural design variants. Again they are introduced in fuzzy structural analysis and fuzzy probabilistic safety assessment to obtain the assigned fuzzy result values. After having checked the complete permissibility of the fuzzy results the design variants are compared according to value and uncertainty of the assigned structural responses and safety levels.

The values of structural responses and safety levels are computed by applying defuzzification methods [26]. Thereby the centroid method and the defuzzification algorithms after Chen [27]and Jain [28] are proposed, whereby the Chen/Jain algorithms permit to weigh smaller or larger values e.g. depending on their importance. The distance between the defuzzified result value $\mathrm{z_{j0}}$ and the initially stated permissible value *perm*_$\mathrm{z_j}$ serves as the first criterion to assess the fuzzy result. The design variant that yields the greatest governing distance $perm\_\mathrm{z_j}-\mathrm{z_{j0}}$ is assessed as being the best one according to the first criterion (presumed that $\mathrm{z_{j0}} \leq perm\_\mathrm{z_j}$ is required).

$$perm\_\mathrm{z_j}-\mathrm{z_{j0}} \Rightarrow \text{Max} \tag{1}$$

The uncertainty of the fuzzy results is assessed on the basis of Shannon's entropy [7, 29]. The fuzzy result $\tilde{\mathrm{z}}_\mathrm{j}$ yields the entropy

$$\mathrm{H}(\tilde{\mathrm{z}}_\mathrm{j}) = -\mathrm{k} \cdot \int_{\mathrm{z_j} \in \tilde{\mathrm{z}}_\mathrm{j}} \Big[\mu(\mathrm{z_j}) \cdot \ln\Big(\mu(\mathrm{z_j})\Big) + \Big(1-\mu(\mathrm{z_j})\Big) \cdot \ln\Big(1-\mu(\mathrm{z_j})\Big)\,\mathrm{dz_j}\Big] \ ; \ \mathrm{k} > 0 \tag{2}$$

as a measure of the absolute uncertainty of $\tilde{\mathrm{z}}_\mathrm{j}$ [22]. This is used as the second criterion when searching for a preferably robust structural design. As the robustness can not be measured in absolute terms a relative sensitivity measure is introduced. If the uncertainty of the fuzzy result $\tilde{\mathrm{z}}_\mathrm{j}$ takes low values in relation to the uncertainty of the fuzzy input values $\tilde{\mathrm{x}}_\mathrm{i}$ the assigned structural design is considered as being robust. This means that moderate fluctuations of input values within the considered uncertainty only lead to small fluctuations in structural response and safety levels. The relative sensitivity measure is defined by

$$\mathrm{B}(\tilde{\mathrm{z}}_\mathrm{j}) = \sum_\mathrm{i} \frac{\mathrm{H}(\tilde{\mathrm{z}}_\mathrm{j})}{\mathrm{H}(\tilde{\mathrm{x}}_\mathrm{i})} \tag{3}$$

Thereby a low value B denotes a low sensitivity, i.e. a high robustness of structural design. The design variant that yields the lowest governing sensitivity $\mathrm{B}(\tilde{\mathrm{z}}_\mathrm{j})$ is assessed as being the best one according to the second criterion.

$$\mathrm{B}(\tilde{\mathrm{z}}_\mathrm{j}) \Rightarrow \text{Min} \tag{4}$$

In general the requirements according to Eqns. (1) and (4) cannot be satisfied simultaneously, a multiple solution is obtained. Depending on the particular design problem concerned the criterions may be weighted by different factors assigned to different fuzzy results and combined to only one objective function.

$$\sum_{\mathrm{j}} \mathrm{a_j} \cdot \frac{perm\_\mathrm{z_j} - \mathrm{z_{jo}}}{perm\_\mathrm{z_j}} - \sum_{\mathrm{j}} \mathrm{b_j} \cdot \frac{\mathrm{B}(\tilde{\mathrm{z}}_\mathrm{j})}{\mathrm{z}_{\mathrm{j},\alpha=1}} \Rightarrow \mathrm{Max} \tag{5}$$

This yields a compromise solution that meets the requirements in the particular case at best.

4 Example

The presented concept of uncertain structural design is demonstrated for the plane reinforced concrete frame shown in Fig. 4. The system is modeled using three bars. Fifty integration increments are chosen for each bar and each cross-section is subdivided into 60 layers. The geometrically and physically nonlinear analysis is carried out using the material laws for reinforcement steel and concrete after OETES (see [30]). Tension stiffening and the effects of stirrup reinforcement are accounted for in the concrete material law.

The loading process is comprised of dead weight, horizontal load $\mathrm{P_H}$, vertical nodal loads $\nu \cdot \mathrm{P_{V0}}$ and the line load $\nu \cdot \mathrm{p_0}$. After applying dead weight the horizontal load $\mathrm{P_H}$ is introduced; $\mathrm{P_{V0}}$ and $\mathrm{p_0}$ are finally increased incrementally using the load factor ν.

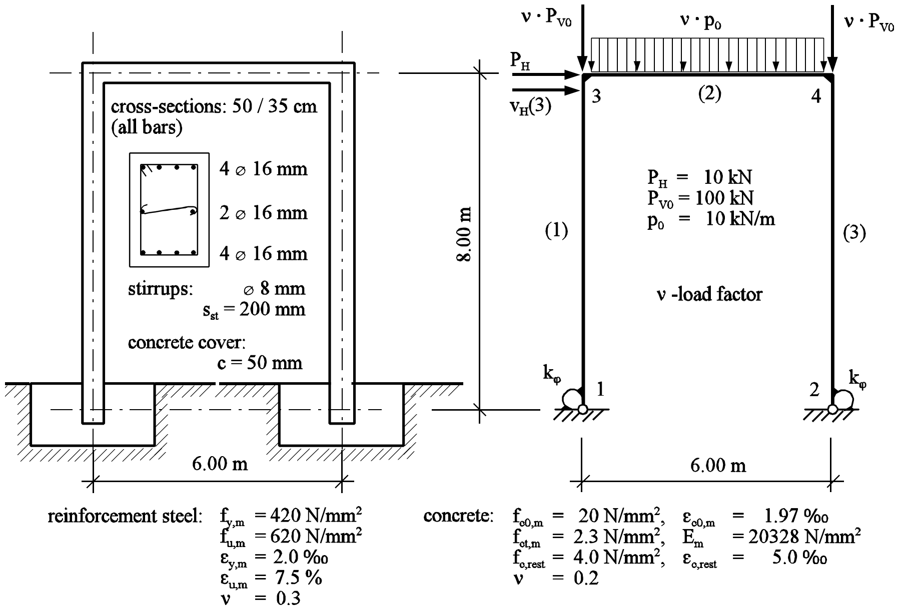

Figure 4: Reinforced concrete frame (plane); structure, cross-sections, materials, statical system with loading

The load factor and the rotational spring stiffness are chosen to be fuzzy design values. In the first step these structural parameters are modeled as fuzzy triangular numbers.

$$\tilde{\nu} = <5.5, 5.9, 6.7> \tag{6}$$

$$\tilde{\mathrm{k}}_{\varphi} = <5.0, 9.0, 13.0> \ [\mathrm{MNm/rad}] \tag{7}$$

Fuzzy result value from the fuzzy structural analysis is the horizontal displacement $\tilde{\mathrm{v}}_{\mathrm{H}}(3)$ of node 3. The investigation is carried out for the α-levels $\alpha_1 = 0.00$, $\alpha_2 = 0.144$, $\alpha_3 = 0.25$, $\alpha_4 = 0.50$, $\alpha_5 = 0.75$ and $\alpha_6 = 1.00$, 75 points in the space of the fuzzy input values are processed. Fig. 5 shows the fuzzy load-displacement dependency for α_1 and α_6 and the fuzzy result $\tilde{\mathrm{v}}_{\mathrm{H}}(3)$. For $\alpha_1 = 0.00$ global system failure occurs before the attainment of $\nu_{\alpha_1 \mathrm{r}} = 6.7$; the search for the maximum of $\mathrm{v}_{\mathrm{H}}(3)$ yields the result $\mathrm{v}_{\mathrm{H}}(3)_{\alpha_1 \mathrm{r}} \rightarrow \infty$ on this α-level.

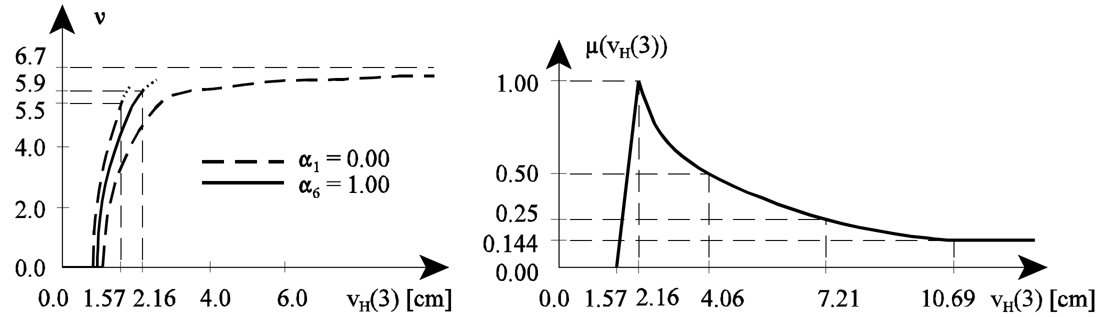

Figure 5: Fuzzy load-displacement dependency for $\tilde{\mathrm{v}}_{\mathrm{H}}(3)$ and $\alpha_1 = 0.00$ and $\alpha_6 = 1.00$; fuzzy result $\tilde{\mathrm{v}}_{\mathrm{H}}(3)$

Requirements concerning serviceability are used to define the permissible displacement $perm\_\mathrm{v}_{\mathrm{H}}(3) = 4.0$ cm. Thus the fuzzy result $\tilde{\mathrm{v}}_{\mathrm{H}}(3)$ possesses both permissible and non-permissible parts. In the space of the fuzzy input values 62 permissible and 13 non-permissible points are identified.

The two sets of points are analyzed one after the other by applying fuzzy cluster analysis. A suitable number of clusters is determined on the basis of the mentioned quality indexes. By considering both the separation degree and the normalized partition coefficient the partitioning consisting of three permissible clusters is assessed as being favorable. Clustering on the α-level $\alpha = 0.30$ (as minimum cluster membership of elements) yields the largest permissible clusters, see Fig. 6. Two of these clusters (C2 and C3) do not overlap non-permissible clusters and thus are capable for generating modified fuzzy input values.

From the cluster result two sets of modified fuzzy input values are generated. The supports are defined using the cluster borders and the cluster centers are taken as mean

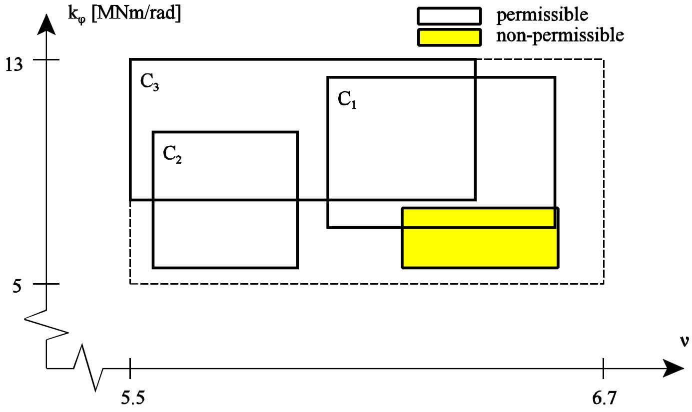

Figure 6: Clustering of the space of the fuzzy input values

values. This yields the fuzzy triangular numbers

$$\tilde{\nu}_{\mathrm{C2}} = < 5.56, 5.74, 5.92 > \tag{8}$$
$$\tilde{\mathrm{k}}_{\varphi \mathrm{C2}} = < 5.6, 8.0, 10.4 > \ [\mathrm{MNm/rad}] \tag{9}$$

according to cluster C2 and

$$\tilde{\nu}_{\mathrm{C3}} = < 5.5, 5.94, 6.38 > \tag{10}$$
$$\tilde{\mathrm{k}}_{\varphi \mathrm{C3}} = < 8.0, 10.5, 13.0 > \ [\mathrm{MNm/rad}] \tag{11}$$

determined by cluster C3. The assigned results from the repeated fuzzy structural analysis are shown in Fig. 7. Both fuzzy results meet the requirement regarding the permissible displacement. Defuzzifying the results after Jain [28] and computing the relative sensitivity measure lead to

- cluster C_2: $\mathrm{v}_{\mathrm{H0}}(3)$ = 2.49 cm, B = 3.81
- cluster C_3: $\mathrm{v}_{\mathrm{H0}}(3)$ = 2.54 cm, B = 2.03

As the fuzzy input values according to cluster C_3 yield a lower defuzzified displacement as well as a lower sensitivity than these from cluster C_2 the modified fuzzy input values from Eqns. (10) and (11) are chosen as being the fuzzy structural design parameters, which have been searched for. These fuzzy structural design parameters have to be ensured by dimensioning e.g. geometrical properties, whereby the remaining uncertainty initialized by non-controllable parameters (e.g. soil stiffness) must not exceed the designed fuzziness. If these requirements cannot be complied with, an improved solution could be derived by merging cluster C_3 and a part of cluster C_1.

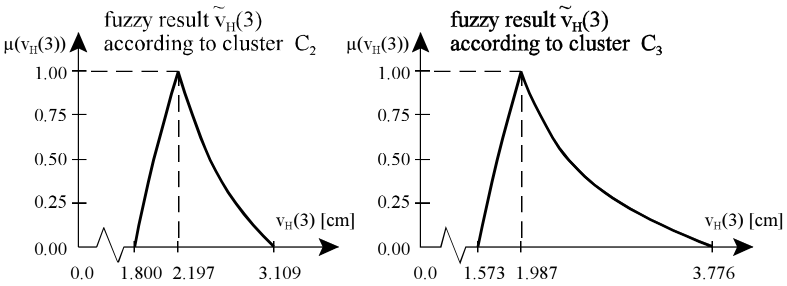

Figure 7: Fuzzy results $\mathrm{v_H}(3)$ according to the alternative sets of permissible fuzzy input values

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