WCCM V
Fifth World Congress on
Computational Mechanics
July 7–12, 2002, Vienna, Austria
Eds.: H.A. Mang, F.G. Rammerstorfer,
J. Eberhardsteiner

Fuzzy Randomness - Towards a new Modeling of Uncertainty

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Key words: fuzzy random variables, fuzzy random functions, fuzzy probabilistic structural analysis, safety assessment of structures

Abstract
In the paper a new concept of modeling uncertainty is presented, based on the theory of fuzzy random variables. By this means, a super-ordinate uncertainty model is made available which includes the models developed so far, based on random variables and fuzzy variables as special cases.

The fuzzy random variables are accounted for as input values in the fuzzy probabilistic structural analysis and the fuzzy probabilistic safety assessment of structures. The method of fuzzy probabilistic structural analysis is demonstrated by way of an example.

1 Introduction

Using fuzzy random variables it is possible to mathematically describe uncertainty characterized by fuzzy randomness. Fuzzy randomness arises when random variables – e.g. as a result of changing boundary conditions – cannot be observed with exactness. Fuzzy random variables may also be interpreted as fuzzified random variables, as the random event can only be observed in an uncertain manner.

Uncertain structural parameters, which partly exhibit random properties, but may not be modeled as random variables without an element of doubt, are described using fuzzy probability distributions. To generate the fuzzy probability distributions statistical methods may be applied and extended to take account of fuzziness in the statistical data.

For the analysis of a structure with the aid of a crisp (or uncertain) algorithm and with fuzzy random variables (or random variables) as input values as well as fuzzy values as model parameters, a fuzzy probabilistic structural analysis is introduced. The combination of a Monte Carlo Simulation with fuzzy probabilistic structural analysis permits the simultaneous consideration of different types of uncertainty: randomness, fuzzy randomness and fuzziness.

Many material, geometrical and loading parameters contain fuzzy random fluctuations in time and space. In order to mathematically describe these parameters fuzzy random functions are introduced, which are accounted for in the structural analysis with the aid of a Fuzzy Stochastic Finite Element Method (FSFEM).

The concept of fuzzy random variables is already applied in the safety assessment of structures, for example as the Fuzzy First Order Reliability Method (FFORM)

2 Remarks on uncertainty

The system behavior and the safety of structures may only be realistically assessed provided all input data are appropriately described and a realistic computational model is implemented. Input and model parameters are often only available in the form of uncertain parameters.

In the case of uncertain parameters, samples containing a limited number of sample elements are often only available, from which a formal mathematical description of uncertainty must be developed. The test theory of classical statistics permits the testing of a sample for randomness. If the sample does not exhibit the property of randomness, other uncertainty models such as, e.g. fuzzy randomness, must be adopted.

For the specific sample shown in Fig. 1 an assessment must be made in relation to the uncertainty to be assigned. The sample elements describe the tensile strength of glass filament yarn. The sample is evaluated on the basis of estimation and test theory. In order to examine the random properties of the sample shown in Fig. 1, parameter-free tests are carried out. The null hypothesis H_0, "the sample elements are random", is rejected with a level of significance of 0.987. The test of homogeneity for the null hypothesis H_0, "the two sub-samples containing elements 1–55 and 56–110 respectively, are derived from the same distribution", is rejected with a significance level of 1.0. With the aid of goodness-of-fit tests, the type of probability distribution of the sample is sought. The KOLMOGOROV SMIRNOV goodness-of-fit test yields a rejection certainty of 0.0 for the normal distribution as well as the logarithmic normal distribution. The unique assignment of a distribution type is hence not possible. The i.i.d. paradigm (i.i.d. = identically independently distributed) of statistics is not fulfilled.

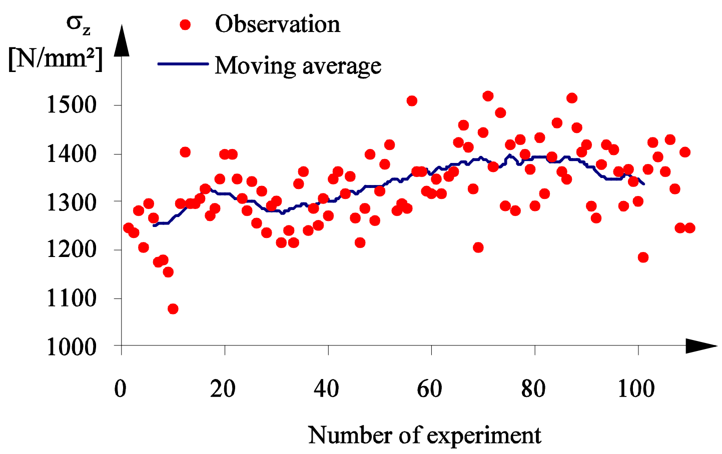

Figure 1: Tensile strenght of glass filament yarn NEG-ARG 620-01

Informal uncertainty, e.g. for defining the distribution type of a universe, does not always permit a description of uncertainty by randomness alone. This is illustrated by a numerical study. A universe is considered with a theoretically exact normal distribution with the parameters E(X) = 3 and VAR(X) = 1. 40,000 samples, each containing n = 100 elements, are considered. Although the distribution type for the universe is known, a normal distribution and a logarithmic normal distribution are assumed as alternatives for the chosen samples. For each sample the parameters of the assumed normal distribution and logarithmic normal distribution are estimated by means of the maximum likelihood method. The expected values E(X) and the variances VAR(X) of the 40,000 estimated distributions are computed and shown in Fig. 2 in the form of a cumulative frequency curve. Although the samples are drawn under constant conditions, it is apparent that significant informal uncertainty exists when the distribution type of a universe cannot be correctly determined.

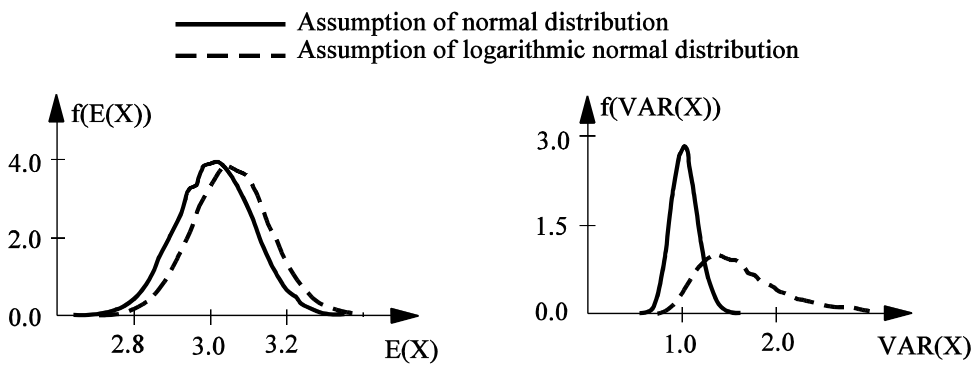

Figure 2: Cumulative frequency curve of the expected value E(X) and the variance VAR(X) assumed different types of probability distribution functions

Particularly in civil engineering, it cannot be presumed that measurements for determining parameters (drawing of samples) are carried out under constant conditions. Material, geometrical and loading parameters are affected by a multitude of variable factors. This leads to non-compliance with the underlying i.i.d. paradigm. Even when applying BAYES statistics as a possible means of introducing additional information via a universe, the fulfillment of the i.i.d. paradigm for the sample variables of several samples is disputed. For it is assumed that the distribution of the universe, from which different samples are sequentially drawn, remains constant. The assumed a priori distribution may significantly affect the results and may lead to non-negligible errors.

In the case of non-constant reproduction conditions, a limited number of samples, uncertain or non-numerical data, or non-compliance with the i.i.d. paradigm, significant uncertainty exists, which must be described by extended uncertainty models. The modeling of uncertainty as fuzzy randomness leads to a generalized uncertainty model containing the special cases "fuzziness" and "randomness". The mathematical description of fuzzy randomness is based on the theory of fuzzy random variables.

3 Fuzzy random variables

The underlying concepts and definitions relating to the theory of fuzzy random variables are expounded in [1], [2] and [3]. Fuzzy random variables are defined by an extension of the axiomatic probability concept after KOLMOGOROV. The probability space $[\underline{\mathbf{X}}; \mathfrak{S}; P]$ is thereby extended by the dimension of fuzziness; the uncertain measure probability remains defined over n-dimensional Euclidian space $\mathbb{R}^n$.

A *fuzzy random variable* $\underline{\tilde{X}}$ is the fuzzy result of the uncertain mapping

$$\Omega \ \tilde{\rightarrow}\ \mathbf{F}(\mathbb{R}^n) \tag{1}$$

with $\mathbf{F}(\mathbb{R}^n)$ as the set of all fuzzy numbers in $\mathbb{R}^n$. Each ordinary random variable $\underline{X}$ (without fuzziness) on $\underline{\mathbf{X}}$, which is completely contained in $\underline{\tilde{X}}$, is referred to as an original of $\underline{\tilde{X}}$. The fuzzy random variable $\underline{\tilde{X}}$ is the *fuzzy set of all possible originals* $\underline{X}$, contained in $\underline{\tilde{X}}$. The fuzzy random variable $\underline{\tilde{X}}$ may be described mathematically by a fuzzy probability distribution function $\tilde{F}(x)$.

The fuzzy probability distribution function $\tilde{F}(x)$ of $\underline{\tilde{X}}$ is the set of probability distribution functions of all originals $\underline{X}_j$ of $\underline{\tilde{X}}$ with the membership values $\mu(F(\underline{x}))$, see Fig. 3. The quantification of fuzziness by fuzzy parameters leads to the description of the fuzzy probability distribution function $\tilde{F}(x)$ of $\underline{\tilde{X}}$ as a function of the fuzzy bunch parameter $\underline{\tilde{s}}$.

$$\tilde{F}(\underline{x}) \ = \ F(\underline{\tilde{s}},\ \underline{x}) \tag{2}$$

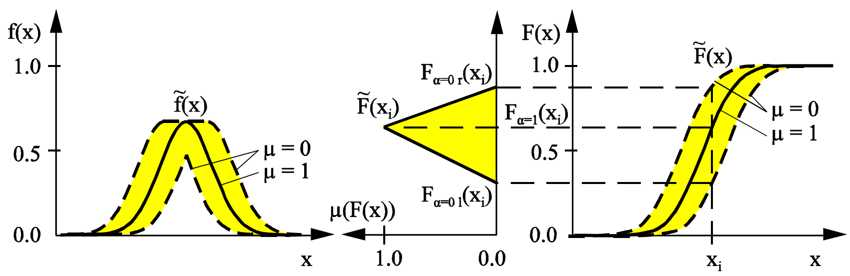

Figure 3: Fuzzy probability density function $\tilde{f}(x)$ and fuzzy probability distribution function $\tilde{F}(x)$ of an one-dimensional continuous fuzzy random variable

For the purposes of numerical evaluation, α-discretization is advantageously applied.

$$F(\underline{\tilde{s}},\ \underline{x}) = \left\{F_\alpha(\underline{x});\ \mu(F_\alpha(\underline{x}))\ |F_\alpha(\underline{x}) = [F_{\min,\alpha}(\underline{x});\ F_{\max,\alpha}(\underline{x})],\ \mu(F_\alpha(\underline{x})) = \alpha,\ \forall\alpha\ \in\ (0,\ 1]\right\} \tag{3}$$

with $F_{\min,\alpha}(\underline{x}) \ = \inf\ \{\ F(\underline{s}, \underline{x})|\ \underline{s} \in \underline{s}_\alpha\ \}$
$F_{\max,\alpha}(\underline{x}) = \sup\ \{\ F(\underline{s}, \underline{x})|\ \underline{s} \in \underline{s}_\alpha\ \}$

4 Fuzzy random functions

Uncertain parameters (e.g. material parameters, loading or boundary conditions), which are mainly dependent on time τ or the spatial coordinates $\underline{\Theta} = \{\Theta_1,\ \Theta_2,\ \Theta_3\}$, and to which the uncertain characteristic fuzzy randomness may be assigned, may be quantified using fuzzy random functions. The time and spatial coordinates are lumped together in the vector $\underline{t} = \{\tau,\ \Theta\}$. A fuzzy random function then becomes a family of fuzzy random variables $\underline{\tilde{X}}(\underline{t}) = \{\underline{\tilde{X}}(\underline{t},\ \omega),\ \underline{t} \in \mathbf{T}\}$ over a joint fuzzy probability space $[\underline{\mathbf{X}};\ \tilde{\mathfrak{S}};\ \tilde{P}]$. A fuzzy random function $\underline{\tilde{X}}(\underline{t}) = \{\underline{\tilde{X}}(\underline{t},\ \omega),\ \underline{t} \in \mathbf{T}\}$ is generated by the uncertain mappings of $T \times \Omega$ on $\mathbf{F}(\mathbb{R}^n)$. An example of a fuzzy random function without randomness is a fuzzy function [9]. An example of a fuzzy random function without fuzziness is an ordinary random function, which is referred to as the original function of the fuzzy random function. For each $\underline{t} \in \mathbf{T} \in \mathbb{R}^n$, $\underline{\tilde{X}}(\underline{t})$ is a fuzzy random variable in the fuzzy probability space $[\underline{\mathbf{X}};\ \tilde{\mathfrak{S}};\ \tilde{P}]$.

With the aid of α-discretization a fuzzy random function may be formulated as a set of α-level sets of ordinary random functions

$$\underline{\tilde{X}}(\underline{t}) = \left\{\underline{X}_\alpha(\underline{t});\ \mu(\underline{X}_\alpha(\underline{t}))\ |\underline{X}_\alpha(\underline{t}) = [\underline{X}_{\min,\alpha}(\underline{t});\ \underline{X}_{\max,\alpha}(\underline{t})],\ \mu(\underline{X}_\alpha(\underline{t})) = \alpha,\ \forall\alpha \in (0,\ 1]\right\} \quad (4)$$

For a fuzzy random function, which is solely dependent on spatial coordinates, the term fuzzy random field is adopted. In the case of time-dependency, the term fuzzy random process is adopted. The definitions of fuzzy random functions is based on [4].

For the fuzzy random function, fuzzy probability distribution functions may be generated as a fuzzy set of the probability distribution functions of the original functions. Analogous to Eqn. (3), a representation dependent on fuzzy bunch parameters is then possible. The properties of fuzzy random functions regarding their randomness may be derived from the theory of random functions. A fuzzy random function is e.g. strictly stationary, when the moments of all orders of the fuzzy probability distribution function are invariant relative to a displacement in the vector $\underline{t}$.

5 Uncertain data analysis

From the wide-ranging field of uncertain data analysis, the generation of fuzzy probability distribution functions is considered here in relation to the modeling of fuzzy randomness. These functions are formulated according to Eqn. (2) in relation to fuzzy bunch parameters. Fuzzy bunch parameters may constitute all functional parameters and/or parameters which describe the functional type.

The methods developed here for uncertain data analysis for the fuzzy evaluation of statistical inference are based on classical as well as modern statistics; bootstrap methods are also included [5]. Moment estimators and maximum likelihood estimators are applied for point and interval estimations of the distribution parameters. Assumed distribution types are assessed with the aid of goodness-of-fit tests. Nonparametric tests for assessing samples (run test, test of homogeneity) are also applied. Two concepts were developed in order to take account of informal uncertainty.

Fuzzy-Parameter estimation
Under the assumption of a functional type for the fuzzy probability distribution function, the estimation problem reduces to the determination of distribution parameters. These are modeled as fuzzy numbers. It is suggested here that the mean value of the fuzzy parameters is defined by point estimation and the α-level set by interval estimation for a prescribed confidence level. This approach is shown in Fig. 4 for the example of a fuzzy triangular number.

$$\hat{\theta} = \Theta\,(x_1,\ \ldots,\ x_n)$$

$$P_\theta[T_u\,(X_1,\ \ldots,\ X_n) < \theta < T_o\,(X_1,\ \ldots,\ X_n)] = \gamma$$

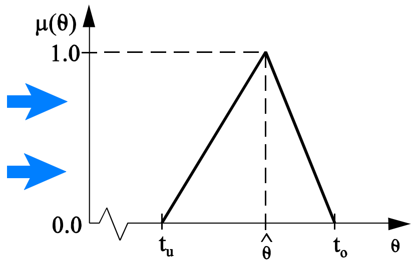

Figure 4: Determination of a fuzzy distribution parameter

Non-parametric estimation of the fuzzy probability distribution
This approach makes it possible to avoid the generally informal uncertain assumption of a distribution type insofar as the empirical distribution function is directly fuzzified and the fuzzy probability distribution function is directly constructed. The method is based on the principle that the empirical distribution function is comprised of binomially-distributed random variables. Applying the method after CLOPPER and PEARSON, confidence intervals for binomially-distributed random variables are determined [5]. These are used to fuzzify the functional values of the fuzzy probability distribution function.

6 Fuzzy probabilistic structural analysis

The aim of fuzzy probabilistic structural analysis is to map the fuzzy probabilistic input values and fuzzy model parameters of the result values with the aid of a crisp (or uncertain) analysis algorithm M $(\tilde{M})$. The results $\tilde{Z}_i \in \underline{\tilde{Z}}$ of the fuzzy probabilistic structural analysis are fuzzy random variables. Separation of fuzziness in the bunch parameter vector $\underline{\tilde{s}}$ leads to

$$\underline{\tilde{Z}} = \left\{\tilde{Z}_1,\ \ldots,\ \tilde{Z}_i,\ \ldots,\ \tilde{Z}_m\right\} = \tilde{M}(\underline{\tilde{X}}) = M(\underline{\tilde{s}},\ \underline{X}) \tag{5}$$

In the following, a physically nonlinear analysis algorithm for general RC folded-plate structures according to [11] is applied as the deterministic fundamental solution. By means of the realistic simulation algorithm it is possible to compute complex loading processes on an incremental-iterative basis, taking account of all essential reinforced concrete nonlinearities.

The fuzzy probabilistic structural analysis is performed with the aid of a 3-stage analysis algorithm. With the aid of α-level optimization [8] the particular elements (realizations) of the fuzzy set $\underline{\tilde{s}}$ (Eqn. (5)) in the space of the fuzzy bunch parameters are determined, which define the largest and smallest value of the α-level set of results $\tilde{Z}_i$ on each α-level. With each element of the fuzzy set $\underline{\tilde{s}}$ an original of the fuzzy random variable is selected. The results $\tilde{Z}_i$ are comprised of a fuzzy expected value, a fuzzy variance or fuzzy quantiles of structural responses such as displacements, deformations or internal forces. Fuzzy probability distribution functions for these result values may be approximated by the set of fuzzy quantile values.

An efficient probabilistic fundamental solution is applied as the mapping operator for computing the results $\tilde{Z}_i$ based on the elements of $\underline{\tilde{s}}$. Within the framework of the probabilistic fundamental solution the deterministic nonlinear structural analysis is carried out repeatedly.

A suitable instrument for determining the probabilistic fundamental solution of complex structures computed using a realistic mathematical-mechanical model is the Monte Carlo Simulation [7]. Depending on the required accuracy regarding the realistic description of physical behavior or the available statistical information, the randomness may be quantified using stationary, homogeneous or non-homogeneous, one-dimensional or multi-dimensional, univariate or multivariate, Gaussian or non-Gaussian models. If the Monte Carlo Simulation is thereby coupled with the nonlinear algorithms implemented in the deterministic structural analysis, it is only possible to solve this problem with a great deal of numerical effort. Suggestions to reduce the computational effort involved are e.g. Quasi-Monte Carlo Methods.

By applying a deterministic fundamental solution based on FE algorithms it is possible to determine fuzzy random functions; this leads to the *Fuzzy Stochastic Finite Element Method (FSFEM)*. The fuzzy random functions are thereby discretized into correlated fuzzy random variables. The correlation between the discretized fuzzy random variables, which is often specified without experimental verification, may be modeled as a fuzzy value.

The FSFEM approach is demonstrated by way of an example. The reinforced concrete folded plate structure shown in Fig. 5 is computed under consideration of the governing nonlinearities of the reinforced concrete and fuzzy randomness associated with the concrete compressive strength and the superficial load.

The dimensions for every plane of the folded plate structure in $\mathbb{R}^2$ are crisp. The system is meshed using 48 hybrid finite elements with assumed stress distribution. Each element, with an overall thickness of 10 cm, was modeled using 7 equidistant concrete layers, with two smeared mesh reinforcement layers each on the upper and lower surfaces (see Fig. 5).

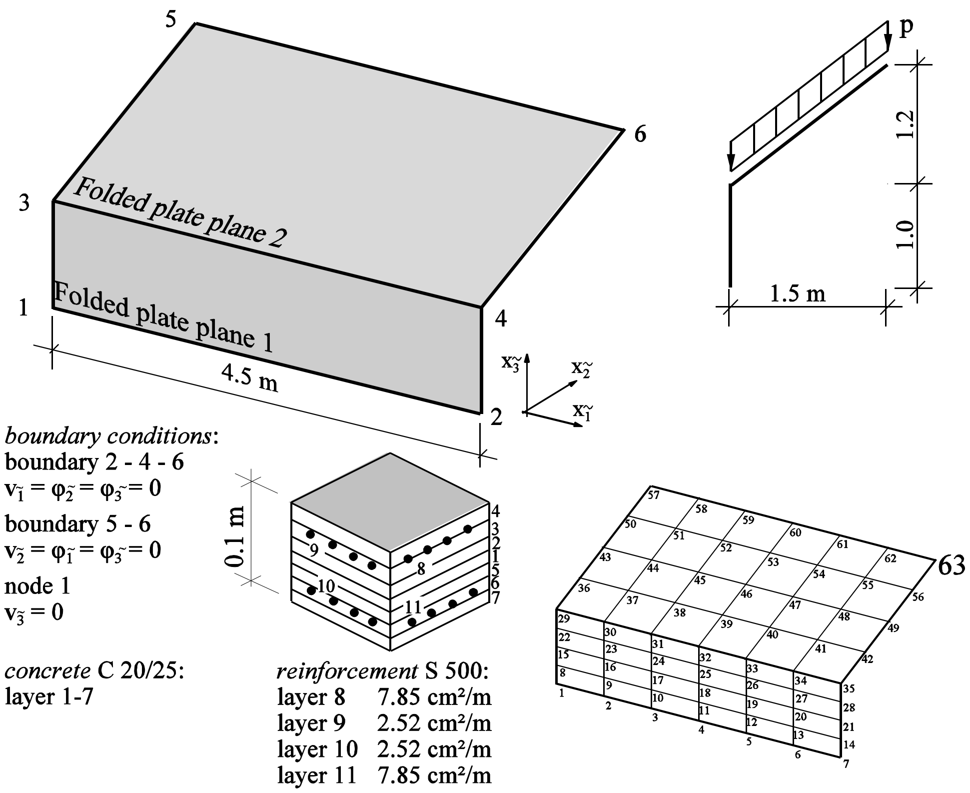

Figure 5: Geometry, finite element model

The concrete material law according to KUPFER/LINK and a bilinear material law for reinforcement steel were applied. Tensile cracks in the concrete were accounted for in each element on a layer-to-layer basis according to the concept of smeared fixed cracks. The superficial load with randomness was increased incrementally up to the service load. Selected result values were computed at this stage of loading. Under consideration of the fuzzy randomness of the uncertain input values, the result values are also fuzzy random variables.

The fuzzy randomness of the concrete compressive strength is modeled using a stationary homogeneous fuzzy random field in $\mathbb{R}^2$ of the planes of the folded-plate structure. The fuzzy random field is discretized in the centroids of the finite elements. The discretization variant is chosen under consideration of the special properties of the applied nonlinear FE algorithm. The following relationship between the tensile and compressive strength of concrete is adopted

$$\tilde{\beta}_t = 0.092\ \tilde{\beta}_c \tag{6}$$

The principal stress criterion (crack criterion) is evaluated in the element centroid (discretization point). A perfect correlation is assumed between the compressive strength and the tensile strength as well as between the strengths of the elements of the layers in a finite element. The correlation of the discretized variables of a folded-plate plane is taken into account by prescribing the fuzzy correlation length according to Fig. 6. The discretized fuzzy random variables of different folded-plate planes are not correlated. The fuzzy mean value and the crisp standard deviation are the same for all discrete fuzzy random variables.

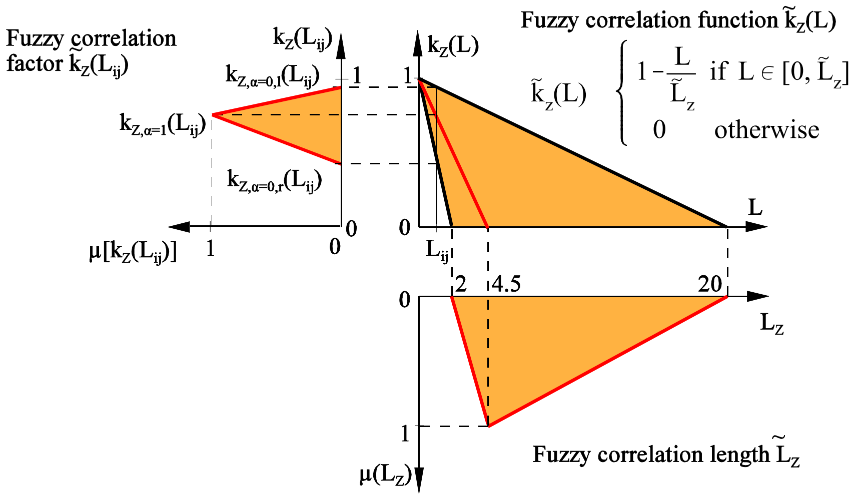

Figure 6: Determination of the fuzzy correlation factor for the spatial distance L_{ij} of two discrete fuzzy random variables $\tilde{X}_i$ and $\tilde{X}_j$

The superficial load p is modeled as an original of a perfect correlated fuzzy random field, i.e. as an ordinary random variable. The parameters are listed in Tab. 1.

α-level-optimization is applied in the fuzzy probabilistic structural analysis; in this case the α-levels $\alpha = 0$ and $\alpha = 1$ are chosen. The fuzzy expected value of the displacement is computed at node 63 in the x_3 direction. The fundamental solution is obtained in each case by a direct Monte Carlo Simulation based on 100 sample elements, from which the expected value is estimated. The approximated membership function of the fuzzy expected value of the displacement is shown in Fig. 7.

Table 1: Parameters of the fuzzy random functions.

	concrete compressive strength β_c [N/mm^2]	superficial load p [kN/m^2]
model	fuzzy random function	random function
type of distribution	normal distribution	normal distribution
expected value	$< 24.5 ; 25.5 ; 26.5 >$	6.0
standard deviation	1.5	0.3
correlation length	$< 2.0 ; 4.5 ; 20.0 >$	∞

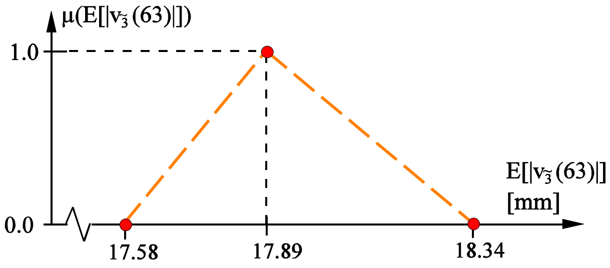

Figure 7: Fuzzy expected value of the fuzzy random displacement $v_{\tilde{3}}$ of node 63

7 Fuzzy probabilistic safety assessment

The aim of the fuzzy-probabilistic safety concept is to determine and assess the safety level of structures under consideration of fuzzy random variables. The result is the fuzzy failure probability or the fuzzy reliability index. The fuzziness of the computed safety level characterizes the new quality of the safety assessment compared with customary probabilistic methods.

The fuzzy failure probability is a measure indicating that the uncertain loading (external load $\tilde{S}$) acting on the structure is larger than the uncertain loadability (resistance $\tilde{R}$)

$$\tilde{P}_f = P(\tilde{R} - \tilde{S}) < 0 \qquad (7)$$

The loading and loadability are described by fuzzy random variables, random variables and fuzzy variables. Together with ordinary random variables, the fuzzy random variables form the space of the basic variables $\tilde{X}_i$. Fuzzy variables are interpreted as model parameters of the uncertain structural model. In the space of the basic variables the survival region is separated from the failure region by the limit state function. According to the selected failure criterion it is possible to define different limit states (limit state of serviceability, limit state of load-bearing capacity). Uncertain computational models with fuzzy model parameters lead to a fuzzy limit state function.

The fuzzy failure probability is obtained by integrating the fuzzy probability joint density function over the fuzzy failure region

$$\tilde{P}_f = \int_{\underline{x} | \tilde{g}(\underline{x}) < 0} \dots \int \tilde{f}_X(x_1, x_2, \dots, x_n)\, dx_1\, dx_2 \dots dx_n \qquad (8)$$

With the aid of α-discretization the integral is evaluated on an original-to-original basis. The space of the bunch parameters of the fuzzy probability distribution functions is extended by the fuzzy model parameters. Realizations of the bunch parameters yield the joint density function $f_X(\underline{x})$ and the crisp limit state $g(\underline{x}) = 0$

The evaluation of the integral on an original-to-original basis (Eqn. (8)) is carried out using analytical methods, numerical integration, simulation methods and probabilistic approximation methods. The probabilistic approximation methods are further developed in such a way that it is possible to take account of basic variables in the form of fuzzy random variables as well as model parameters in the form of fuzzy variables. The first order reliability theory (FORM) is extended to yield the first order fuzzy reliability theory (FFORM) [10].

Analogous to the method of fuzzy probabilistic structural analysis, α-level optimization is performed in the extended space of the fuzzy bunch parameters (s-space, fuzzy bunch parameters of the fuzzy probability distribution functions and fuzzy model parameters). The optimization targets are the smallest and largest failure probability on each α-level. The originals of the joint density function and the limit state function are known for discrete points in the s-space. The corresponding reliability index is determined using FORM. The result of the α-level optimization is the fuzzy reliability index $\tilde{\beta}$, which may be converted into the fuzzy failure probability $\tilde{P}_f$.

Acknowledgements

Authors gratefully acknowledge the support of the German Research Foundation (DFG) within the framework of Collaborative Research Centre (SFB) 528.

References

[1] H. Kwakernaak, *Fuzzy random variables - I. Definitions and Theorems*, Information Sciences, 15, (1978), pp. 1-29, *Fuzzy random variables - II. Algorithms and Examples for the Discrete Case*, Information Sciences, 17, (1979), pp. 253–278.

[2] M.L. Puri, D. Ralescu, *Fuzzy random variables*, J. Math. Anal. Appl., 114, (1986), pp. 409–422.

[3] Liu Yubin, Qiao Zhong, Wang Guangyuan, *Fuzzy random reliability of structures based on fuzzy random variables*, Fuzzy Sets and Systems, 86, (1997), pp. 345–355.

[4] Guangyuan Wang, Yue Zhang, *The theory of fuzzy stochastic processes*, Fuzzy Sets and Systems, 51, (1992), pp. 161–178.

[5] R. Schlittgen, *Statistische Inferenz*, Oldenbourg Verl., München, Wien, (1996).

[6] G.I. Schuëller (Ed.), *A State-of-the-Art Report on Computational Stochastic Mechanics*, Probabilistic Engineering Mechanics, 12(4), (1997), pp. 197–321.

[7] G.I. Schuëller, *On computational procedurs for processing uncertainties in structural mechanics*, In: *ECCM 2*, Crakow, (2001), CD-ROM.

[8] B. Möller, W. Graf, M. Beer, *Fuzzy structural analysis using α-level-optimization*, Computational Mechanics, 26(6), (2000), pp. 547–565.

[9] B. Möller, M. Beer, W. Graf, J.-U. Sickert, *Fuzzy finite element method and its application*, In: *Trends in computational structural mechanics*, CIMNE, Barcelona, (2001), pp. 529–538.

[10] B. Möller, M. Beer, W. Graf, J.-U. Sickert, *Fuzzy probabilistic methods and its application for safety assessment of structures*. In: *ECCM 2*, Crakow, (2001), CD-ROM.

[11] W. Graf, B. Möller, A. Hoffmann, F. Steinigen, *Numerical simulation of RC structures with textile reinforcement*. In: *Fifth World Congress on Computational Mechanics*, Vienna, (2002), CD-ROM.