Safety Assessment using Fuzzy Theory

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Abstract

Uncertain input parameters may result from "fuzziness", "randomness" or "fuzzy randomness". With the use of fuzzy set theory, uncertain input parameters may be described mathematically as fuzzy variables or fuzzy random variables and may be integrated into safety assessment analysis. With the aid of α-discretization involving the multiple solution of special optimization problems, fuzzy input parameters are mapped onto the uncertain result set. If the deterministic input data are characterized by "fuzziness", the fuzzy results are uncertain outcomes of the structural analysis; safety assessment may then be carried out using possibility theory. If the input parameters exist in the form of fuzzy random variables, the computed fuzzy failure probabilities may be used for safety assessment. A fuzzy 1st-order reliability method (FFORM) is proposed, which is capable of handling fuzzy as well as fuzzy random variables.

Introduction

The safety of structures may be assessed on the basis of different concepts. Simple forms, such as the use of a global safety factor or a semi-probabilistic approach involving partial safety factors are unable to account for (or permit only a very rough estimate of) uncertainties in the input data. These methods are unable to provide a measure of safety. In the case of safety assessment based on probability theory, uncertainties in the input parameters (presupposing large, theoretically infinite, fundamental totalities) are described and analyzed with the aid of mathematical statistics. Probabilistic approximation solutions may be obtained using 1st-order and 2nd-order reliability method (FORM and SORM, respectively). The failure probability P_f and the reliability index ß serve as suitable indicators of the reliability of a structure.

Generally speaking, the available information concerning input parameters is very limited or based purely on estimates provided by experts (also linguistic). The existing uncertainties must therefore be described in their natural form (characteristic state) and

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included in the investigation. The problem in question as well as the type of input parameters thereby govern the basic strategy for solving a particular task. This corresponds to the basic concept of the so-called "toolbox philosophy" (Hung, T.N.; 1997). Besides the probabilistic description of uncertainties, fuzzy modeling as well as convex modeling are also available. This model group is referred to as an "uncertainty triangle" in (Elishakoff, I.; 1995).

The use of fuzzy set theory to account for "fuzziness" and "fuzzy randomness" in the input parameters is demonstrated in the following sections. If *no statistical data material* is available for the input parameters, safety is assessed with the aid of *possibility theory*. *Uncertain statistical information* describing the input parameters is analyzed using the *theory of fuzzy random variables* and applied for safety assessment purposes.

Safety assessment using the possibility theory

Safety assessment using possibility theory is carried out by comparing the *existing* failure possibility $extg\_\Pi_f$ with the *permissible* failure possibility $perm\_\Pi_f$.

In order to compute the existing failure possibility $extg\_\Pi_f$ the uncertain input parameters $\tilde{x}_i$ are fuzzified. With the aid of membership functions $\mu(x_i)$ it is possible to describe fuzzy numbers and fuzzy intervals. Fuzzification permits the processing of objective information (e.g. based on samples) and is also able to take account of subjective estimates by experts. By this means it is possible to realize a mathematical description of uncertain input information in the form of fuzzy input parameters $\tilde{x}_i$.

During fuzzy analysis the fuzzy input parameters are mapped onto the result space by applying the extension principle in combination with the cartesian product (Bothe, H.H.; 1993; Zadeh, L.A.; 1965). When applied to problems in structural analyses, α-discretization is advantageous for the numerical treatment of the fuzzy analysis. The mapping operator is thereby represented by a deterministic algorithm for computing the results in the structural analysis; the problem itself must be solved as an optimization problem. The optimization objective for the result in question (as a functional value of an objective function) is determined by the max-min operator. The application of this method is described e.g. in (Bonarini, A.; Bontempi, G.; 1994). Generalization of the method using a modified evolution strategy for solving optimization problems is given in (Möller, B.; 1997; Möller, B.; Beer, M.; 1997).

Evaluation of the fuzzy results $\tilde{z}_j$ in combination with crisp or uncertain failure functions $\pi(z_j)$ yields the existing failure possibility $extg\_\Pi_f$.

$$extg\_\Pi_f = \sup_{z_j \in Z_j} \min\left(\mu(z_j)\,,\,\pi(z_j)\right) \tag{1}$$

Possibility theory (Bothe, H.H.; 1993; Dubois, D.; Prade, H.; 1986) serves as a basis for the latter. The solution point $\mathbf{x}_L$ corresponding to $extg\_\Pi_f$ is obtained in the input parameter space.

The permissible failure possibility $perm\_\Pi_f$ is determined on the level of 1st-order reliability theory. The safety to be complied with is defined by prescribing a required reliability index *reqd_ß*. For each input parameter the probability of occurrence $\tilde{F}(x_{iL})$ at the solution point $\mathbf{x}_L$ is estimated as a fuzzy value. The "distances" between the possibility scale $\Pi(x_{iL}) = extg\_\Pi_f$ and the probability scales $\tilde{F}(x_{iL})$ at $\mathbf{x}_L$ are introduced as uncertain ratios

$$\tilde{b}(x_{iL}) = \frac{extg\_\Pi_f}{\tilde{F}(x_{iL})} \quad ; i = 1,...,n \tag{2}$$

This enables the position of the uncertain solution point $\tilde{\mathbf{y}}_L$ in standard normal space to be described as the point of intersection between an uncertain hyperbola and the (as a result of the uncertain transformation $X \to \tilde{Y}$) uncertain limit-state surface $h(\tilde{\mathbf{y}}) = 0$. By linearizing the limit-state surface in $\tilde{\mathbf{y}}_L$ it is possible to derive a relationship for the assigned reliability index $\tilde{\beta}$. The prescribed value of *reqd_ß* to be complied with permits the determination of an uncertain permissible solution point $perm\_\tilde{\mathbf{x}}_L$. Assuming that the relationship

$$\tilde{b}(perm\_\tilde{x}_{iL}) \approx \tilde{b}(x_{iL}) \quad ; i = 1,...,n \tag{3}$$

holds, the permissible possibilities of occurrence $\tilde{\Pi}(perm\_\tilde{x}_{iL})$ and a permissible failure possibility $perm\_\tilde{\Pi}_f$ follow from Eqn (2). The calculation of $perm\_\tilde{\Pi}_f$ is carried out in accordance with possibility theory; this provides a basis for an optimistic or pessimistic assessment of the load-bearing capacity. A full description of the method outlined here is presented in (Möller, B.; Beer, M.; Graf, W.; Hoffmann, A.; 1999).

Safety assessment using the theory of fuzzy random variables

The theory of fuzzy random variables forms a mathematical basis for simultaneously accounting for the uncertainties "randomness" and "fuzziness" (Liu, Y.; Qiao Z.; Wang G.; 1997). This approach for handling uncertain input parameters leads to uncertain probability density and probability distribution functions as well as uncertain limit-state functions, and as a result of reliability analysis, also yields fuzzy values for the failure probability $\tilde{P}_f$ and the reliability index $\tilde{\beta}$.

The following models may be used to describe the "fuzziness" of the random variables:

- a distribution function is of the uncertain type (mixed distribution with fuzzy parameters for the individual components),
- the statistical parameters of a distribution function are fuzzy values,
- the limit-state surface is characterized by "fuzziness".

For the steel girder shown in Fig. 1 ($W_{pl} = 366$ cm^3) the distribution function types are known for the point load $x_1 = V$ (extreme value distribution of type I) and the yield stress $x_2 = f_y$ (logarithmic normal distribution). The distribution function for f_y has a minimum value of $x_{0,2} = 199$ N/mm² and an expectation value of $\overline{x}_2 = 288$ N/mm². Due to insufficient samples, the remaining statistical parameters are uncertain. These are included in the investigation as fuzzy triangular values $\tilde{\bar{x}}_1$ = <47; 50; 52> kN, $\tilde{\sigma}_{x1}$ = <4.5; 5.0; 6.0> kN and $\tilde{\sigma}_{x2}$ = <22.0; 26.4; 28.0> N/mm². The failure criterion is system failure. According to 1st-order plasticity theory, the linear limit-state surface is given by

$$g(x_1, x_2) = x_2 - l_1 \cdot l_2 \cdot \left[W_{pl}(l_1 + l_2) \right]^{-1} \cdot x_1 = 0 \quad (4)$$

Safety assessment according to the proposed fuzzy 1st-order reliability method (FFORM) yields the fuzzy reliability index $\tilde{\beta}$ as shown in Fig. 1.

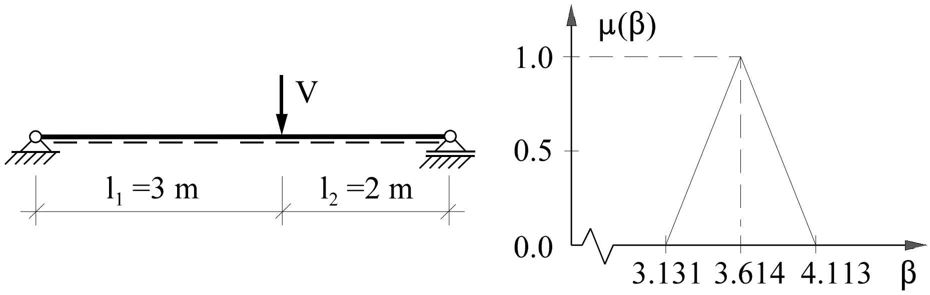

Fig. 1 Loaded steel girder, fuzzy reliability index $\tilde{\beta}$

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